FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS AND STATISTICS
FINAL EXAMINATION
MATH 221 (L 04)

FALL 2003 | Time: 3 hours

I.D. NUMBER	SURNAME	OTHER NAMES

STUDENT IDENTIFICATION

Each candidate must sign the Seating List confirming presence at the examination. All candidates for final examinations are required to place their University of Calgary student I.D. cards on their desks for the duration of the examination. (Students writing mid-term tests can also be asked to provide identity proof.) Students without an I.D. card who can produce an **acceptable** alternative I.D., e.g., one with a printed name and photograph, are allowed to write the examination.

A student without acceptable I.D. will be required to complete an Identification Form. The form indicates that there is no guarantee that the examination paper will be graded if any discrepancies in identification are discovered after verification with the student's file. A student who refuses to produce identification or who refuses to complete and sign the Identification Form is not permitted to write the examination.

EXAMINATION RULES

1. Students late in arriving will not normally be admitted after one-half hour of the examination time has passed.
2. No candidate will be permitted to leave the examination room until one-half hour has elapsed after the opening of the examination, nor during the last 15 minutes of the examination. All candidates remaining during the last 15 minutes of the examination period must remain at their desks until their papers have been collected by an invigilator.
3. All enquiries and requests must be addressed to supervisors only.
4. Candidates are strictly cautioned against:
   (a) speaking to other candidates or communicating with them under any circumstances whatsoever;
   (b) bringing into the examination room any textbook, notebook or memoranda not authorized by the examiner;
   (c) making use of calculators and/or portable computing machines not authorized by the instructor;
   (d) leaving answer papers exposed to view;
   (e) attempting to read other students' examination papers.

   The penalty for violation of these rules is suspension or expulsion or such other penalty as may be determined.
5. Candidates are requested to write on both sides of the page, unless the examiner has asked that the left half page be reserved for rough drafts or calculations.
6. Discarded matter is to be struck out and not removed by mutilation of the examination answer book.
7. Candidates are cautioned against writing in their answer books any matter extraneous to the actual answering of the question set.
8. The candidate is to write his/her name on each answer book as directed and is to number each book.
9. A candidate must report to a supervisor before leaving the examination room.
10. Answer books must be handed to the supervisor-in-charge promptly when the signal is given. Failure to comply with this regulation will be cause for rejection of an answer paper.
11. If a student becomes ill or receives word of domestic affliction during the course of an examination, he/she should report at once to the Supervisor, hand in the unfinished paper and request that it be cancelled. Thereafter, if illness is the cause, the student must go directly to University Health Services so that any subsequent application for a deferred examination may be supported by a medical certificate. An application for Deferred Final Examinations must be submitted to the Registrar by the date specified in the University Calendar.
    Should a student write an examination, hand in the paper for marking, and later report extenuating circumstances to support a request for cancellation of the paper and for another examination, such request will be denied.
12. SMOKING DURING EXAMINATIONS IS STRICTLY PROHIBITED.

Question	Total Marks	Actual Marks
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

CALCULATORS ARE NOT ALLOWED

[10] 1. Find conditions on the constant a so that the following system has (i) no solution, (ii) exactly one solution or (ii) infinitely many solutions.

$$\begin{array}{rcrcrcr} x & + & ay & - & z & = & 1 \\ -x & + & (a-2)\,y & + & z & = & -1 \\ 2x & + & 2y & + & (a-2)\,z & = & 1 \end{array}$$

[10] 2. Let $A = \begin{bmatrix} 2 & 1 & 2 \\ -3 & -1 & -1 \\ 5 & 2 & 1 \end{bmatrix}$.

(a) Find A^{-1} if A is invertible.

(b) Solve the system (you may want to use the result in part (a)):

$$\begin{array}{rcrcrcl} 2x & + & y & + & 2z & = & 1 \\ -3x & - & y & - & z & = & 0 \\ 5x & + & 2y & + & z & = & 1 \end{array}$$

[10] 3. Rookie Frosh is taking a Linear Albegra course at Hay University which has a lecture every day. It is well known that if Rookie skips a lecture then he will not skip the next lecture. However, if he attends one lecture then he is equally likely to skip the next lecture as to attend the next lecture.

(a) If Rookie skips a lecture one day, what is the probability that he skips the lecture three days later?

(b) Find the long run probability that he skips a lecture.

[10] 4. Let $A = \begin{bmatrix} 5 & 6 \\ -3 & -4 \end{bmatrix}$.

(a) Is A diagonaliable? If A is diagonalizable find an invertible matrix P so that $P^{-1}AP = D$ where D is a diagonal matrix.

(b) Compute A^8.

[10] 5. Let $A = \begin{bmatrix} 2 & 1 & 2 \\ -3 & -1 & -1 \\ 5 & 2 & 1 \end{bmatrix}$.

(a) Find $adj\, A$.

(b) Compute $A.adj\, A$.

(c) Find $\det A$.

(d) Can we use Cramer's Rule for the following system? Explain. Do not solve the system.

$$\begin{array}{rcrcrcl} 2x & + & y & + & 2z & = & 1 \\ -3x & - & y & - & z & = & 0 \\ 5x & + & 2y & + & z & = & 1 \end{array}$$

[10] 6. Prove the following statements:

(a) If A and B be the end-points of a diameter of a circle and C is any other point on the circle then the line segment AC and BC are perpendicular.

(b) For any vectors $\overrightarrow{v}$ and $\overrightarrow{w}$, $\|\overrightarrow{v} + \overrightarrow{w}\|^2 + \|\overrightarrow{v} - \overrightarrow{w}\|^2 = 2\left(\|\overrightarrow{v}\|^2 + \|\overrightarrow{v}\|^2\right)$.

[10] 7. For the following, express your answers in the form $a + bi$ where a and b are real numbers.

(a) Compute $(1 - i)^{20}$.

(b) Find all complex numbers z so that $z^3 = -27i$.

[10] 8. Consider the points $A\,(1, 0, -3)$, $B\,(3, 0, -1)$ and $C\,(5, 2, -1)$.

(a) Find the internal angles of the triangle with vertices A, B and C.

(b) Find an equation of the plane containing the points A, B and C.

(c) Find the shortest distance from the point $P\,(1, 2, 3)$ to the plane in part (b).

[10] 9. Let L_1 be the line with equation $[x, y, z]^T = [2, -5, 1]^T + t\,[0, 1, 1]^T$ and L_2 be the line with equation $[x, y, z]^T = [5, 0, 0]^T + s\,[2, 1, 0]^T$.

(a) Are L_1 and L_2 parallel? Explain.

(b) Find the shortest distance between L_1 and L_2, and find a point A on L_1 and a point B on L_2 so that $\left\|\overrightarrow{AB}\right\|$ is the shortest distance between L_1 and L_2.

[10] 10. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation resulted by a reflection in the line $y = x$ followed by a reflection in the x-axis.

(a) Find the matrix of T; that is, find a matrix A so that $T\overrightarrow{v} = A\overrightarrow{v}$ for all $\overrightarrow{v} \in \mathbb{R}^2$.

(b) Is T any of the following: a reflection, a rotation, a projection? Explain.

(c) Is T invertible? If T is invertible, find $T^{-1}\overrightarrow{a}$, where $\overrightarrow{a} = [1, 1]^T$.

End of Examination