MATHEMATICS 221 L04 FALL 2003
MIDTERM EXAMINATION
Thursday, November 6, 2003

I.D.#

I agree that this paper may be placed at the front of the classroom for pick-up.				
Please initial either **YES**		or **NO**		

NO CALCULATORS ALLOWED
ANSWER ALL QUESTIONS
SHOW ALL WORK

LAST NAME_____ **FIRST NAME**_____

[5] **1**. Solve the system:

$$\begin{array}{rcrcrcrcl} x & - & 2y & - & z & + & 3w & = & 1 \\ 2x & - & 4y & + & z & & & = & 5 \\ x & - & 2y & + & 2z & - & 3w & = & 4 \end{array}$$

LAST NAME_____ **FIRST NAME**_____

[5] **2**. Given that $\det \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix} = 5$. Find $\det \begin{bmatrix} a+2x & b+2y & c+2z \\ 3x+4p & 3y+4q & 3z+4r \\ -2p & -2q & -2r \end{bmatrix}$.

[5] **3**. Let $A = \begin{bmatrix} 3 & -1 \\ -2 & 0 \end{bmatrix}$. Find an invertible matrix U so that $UA = R$ where R is the reduced row-echelon form of A and express U as a product of elementary matrices.

LAST NAME_____ **FIRST NAME**_____

[5] **4**. Find all values of x so that the matrix $\begin{bmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{bmatrix}$ is **not** invertible.

[5] **5**. Prove that if $A^3 = 0$ then $I - A$ is invertible and $(I - A)^{-1} = I + A + A^2$.

LAST NAME_____ **FIRST NAME**_____

[5] **6**. Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$. Is A diagonalizable? If A is diagonalizable, find an invertible matrix P and a diagonal matrix D so that $A = PDP^{-1}$.

[5] **7**. For each of the following statements, determine whether the statement is true (T) or false (F). No explanation is needed.

(a) If $A^2 = A$ then $A = 0$ or $A = I$. ☐

(b) If $A^3 = 3I$ then A is invertible. ☐

(c) $\begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$ is an elementary matrix. ☐

(d) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then $adjA = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$. ☐

(e) If A is a square matrix and A is not invertible then 0 is an eigenvalue of A. ☐

LAST NAME_____ **FIRST NAME**_____

[5] **8**. Find all complex numbers z so that $z^3 = -27i$. Express your answers in the form $a+bi$ where a and b are real numbers.