University of Calgary
Faculty of Science
Department of Mathematics and Statistics

Math 221 Fall 2006

Quizzes

1. Find (if possible) values for a so that the given system of equations has (i) no solution, (ii) exactly one solution, (iii) infinitely many solutions.

$$x + 2y - 4z = 4$$

$$3x - y + 13z = 2$$

$$4x + y + a^2 z = a + 3$$

2. Give an example of a system of homogeneous equations which has
   i. The trivial solution only.
   ii. Infinitely many solutions.
   iii. Is it possible for a homogenous system of equations to have no solution? Explain.

3. Find (if possible) values for a so that the given system of equations has (i) the trivial solution only, (ii) infinitely many solutions. When the system has infinitely many solutions, determine all the solutions and stte the rank of the augmented matrix.

$$ax + y + z = 0$$

$$x + y - z = 0$$

$$x + y + az = 0$$

4. Find the matrix A which satisfies the equation given below:

$$\left(3A^T + 2\begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix} \right)^T = \begin{pmatrix} 8 & 0 \\ 3 & 1 \end{pmatrix}$$

5. Find, if possible, values for x, y, z, and w, so that the equation given below is satisfied:

$$\begin{pmatrix} x - y & y - z \\ z - w & w - x \end{pmatrix} = 2\begin{pmatrix} 1 & 1 \\ -3 & 1 \end{pmatrix}$$

6. Find the matrix A which satisfies the equation given below:

$$(2A - 3\begin{pmatrix} 1 & 2 & 0 \end{pmatrix})^T = 3A^T + \begin{pmatrix} 2 & 1 & -1 \end{pmatrix}^T$$

7. Given that $A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$

   a. Determine the inverse of A

   b. Express A as a product of elementary matrices.

University of Calgary
Faculty of Science
Department of Mathematics and Statistics

Math 221 Fall 2006

Quizzes

8. An nxn matrix P is called an idempotent matrix exactly when $P^2 = P$.
Show that P is idempotent if and only if I - 2P is its own inverse.

9. Given that $A = \begin{pmatrix} 1 & -4 \\ 2 & 1 \end{pmatrix}$

a. Express the matrix A as a product of elementary matrices.

b. Express A^{-1} as a product of elementary matrices.

10. Say whether the statement given below is true or false. If the statement is true, prove the assertion. If the statement is false give a counterexample.

"If A and B are both invertible nxn matrices, then $(A^{-1} B)^T$ is invertible."

11. Given that $A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$

a. Determine the inverse of A

b. Express A as a product of elementary matrices.

12. An nxn matrix P is called an idempotent matrix exactly when $P^2 = P$.
Show that P is idempotent if and only if I - 2P is its own inverse.

13. Given that $A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$

a. Determine the inverse of A

b. Express A as a product of elementary matrices.

14. An nxn matrix P is called an idempotent matrix exactly when $P^2 = P$.
Show that P is idempotent if and only if I - 2P is its own inverse.

15. Show that $\det \begin{pmatrix} 1 & x & x^2 & x^3 \\ a & 1 & x & x^2 \\ p & b & 1 & x \\ q & r & c & 1 \end{pmatrix} = (1 - ax)(1 - bx)(1 - cx)$

16. Use Cramer's rule to determine the variable y in the system of equations given below:

$$4x - y + 3z = 1$$

$$6x + 2y - z = 0$$

$$3x + 3y + 2z = -1$$

17. If A is an nxn matrix, show that

$$\det(adj\ A) = (\det A)^{n-1}$$

18. Given that $A = \begin{pmatrix} 1 & x & x^2 & x^3 \\ x & x^2 & x^3 & 1 \\ x^2 & x^3 & 1 & x \\ x^3 & 1 & x & x^2 \end{pmatrix}$, find x so that det A = 0.

19. Given that $A = \begin{pmatrix} 4 & -1 & 3 \\ 6 & 2 & -1 \\ 3 & 3 & 2 \end{pmatrix}$, determine (i) det A (ii) cof A (iii) A^{-1}.

20. Given the matrix $A = \begin{pmatrix} -1 & -3 & 9 \\ -3 & -1 & 9 \\ 0 & 0 & 2 \end{pmatrix}$. Determine the eigenvalues of A and the associated eigenvectors. If A is diagonalizable, determine the matrix P which diagonalizes A.

21. Determine the equation of the straight line which passes through the point A(2,-3,-4) and which is parallel to the straight line which has equation,

$$x = 3 + 4t$$

$$y = -2 + t$$

$$z = 1$$

22. Given the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ -6 & 2 & 6 \\ -2 & 0 & 4 \end{pmatrix}$. Determine the eigenvalues of A and the associated eigenvectors. If A is diagonalizable, determine the matrix P which diagonalizes A.

23. Determine the equation of the straight line which passes through the point A(2,-3,-4) and which is parallel to the straight line which has equation given by:

$$x = 3 - 4t$$
$$y = 5 - t$$
$$z = -5t$$

24. Given the matrix $A = \begin{pmatrix} 2 & -2 & 2 \\ 1 & 0 & 1 \\ 1 & -1 & 2 \end{pmatrix}$. Determine the eigenvalues of A and the associated eigenvectors. If A is diagonalizable, determine the matrix P which diagonalizes A.

25. Determine the equation of the straight line which passes through the point A(2,-3,-4) and the point B(1,-2,-1).