

Practice Problems S4

1. By inspection, find the determinants of the following matrices:

$$(a) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}; (b) \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}; (c) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix};$$
$$(d) \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 4 \\ 2 & -4 & 6 \end{bmatrix}; (e) \begin{bmatrix} 1 & 0 & 4 & 9 \\ -8 & -7 & 12 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 5 & 3 \end{bmatrix}.$$

2. Compute the determinants of the following matrices

$$(a) A = \begin{bmatrix} -2 & 1 & 3 \\ 1 & -7 & 4 \\ -2 & 1 & 3 \end{bmatrix}; (b) A = \begin{bmatrix} 3 & 5 & -2 & 6 \\ 1 & 2 & -1 & 1 \\ 2 & 4 & 1 & 5 \\ 3 & 7 & 5 & 3 \end{bmatrix}.$$

3. Find the inverse of $A = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$ using the adjoint formula.

4. Given $A = \begin{bmatrix} 3 & -1 & 2 \\ 5 & 5 & -2 \\ 1 & 2 & 3 \end{bmatrix}$, find the (1,3)-entry of A^{-1} .

5. For which values of $c \in \mathbb{R}$ is A invertible if $A = \begin{bmatrix} 1 & c & 0 \\ 2 & 0 & c \\ c & -1 & 1 \end{bmatrix}$.

6. Solve the following system by Cramer's rule:

$$(a) \begin{cases} x + 2y = 4 \\ 3x + 7y = 13 \end{cases}; (b) \begin{cases} 3x - 2y + 4z = -3 \\ 5x + 3y + z = 0 \\ 2x + 6y - 5z = 6 \end{cases}.$$

Solutions

1. (a): -1; (b): -3; (c)1; (d): 0; (e):-21

$$2. \text{ (a): } 0; \text{ (b): } \det(A) = \begin{vmatrix} 3 & 5 & -2 & 6 \\ 1 & 2 & -1 & 1 \\ 2 & 4 & 1 & 5 \\ 3 & 7 & 5 & 3 \end{vmatrix} = \begin{vmatrix} 0 & -1 & 1 & 3 \\ 1 & 2 & -1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 1 & 8 & 0 \end{vmatrix} = \begin{vmatrix} -1 & 1 & 3 \\ 0 & 3 & 3 \\ 1 & 8 & 0 \end{vmatrix} =$$

$$\begin{vmatrix} 0 & 9 & 3 \\ 0 & 3 & 3 \\ 1 & 8 & 0 \end{vmatrix} = \begin{vmatrix} 9 & 3 \\ 3 & 3 \end{vmatrix} = -18.$$

3. $A^{-1} = \frac{1}{\det(A)} \text{adj}(A)$, where $\det(A) = -1$ and $\text{adj}(A) = [c_{ij}(A)]^T =$

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & -1 \\ -1 & 1 & -1 \end{bmatrix}^T = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix}. \text{ Therefore, } A^{-1} = \begin{bmatrix} -1 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}.$$

4. A has determinant $\det(A) = 84$. The $(\mathbf{1}, \mathbf{3})$ -entry of A^{-1} is $\frac{1}{\det(A)} c_{31}(A) =$

$$\frac{1}{84}(-1)^{3+1} \begin{vmatrix} -1 & 2 \\ 5 & -2 \end{vmatrix} = -\frac{8}{84} = -\frac{2}{21}.$$

5. A is invertible iff $0 \neq \det(A) = c^3 - c = c(c-1)(c+1)$. So, A is invertible iff $c \in \mathbb{R} \setminus \{1, 0, -1\}$.

$$6. \text{ (a) } x = \frac{\begin{vmatrix} 4 & 2 \\ 13 & 7 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix}} = \frac{28 - 26}{7 - 6} = 2; y = \frac{\begin{vmatrix} 1 & 4 \\ 3 & 13 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix}} = \frac{13 - 12}{7 - 6} = 1.$$

$$\text{(b) } x = \frac{\begin{vmatrix} -3 & -2 & 4 \\ 0 & 3 & 1 \\ 6 & 6 & -5 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & 4 \\ 5 & 3 & 1 \\ 2 & 6 & -5 \end{vmatrix}} = \frac{-21}{-21} = 1; y = \frac{\begin{vmatrix} 3 & -3 & 4 \\ 5 & 0 & 1 \\ 2 & 6 & -5 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & 4 \\ 5 & 3 & 1 \\ 2 & 6 & -5 \end{vmatrix}} = \frac{21}{-21} =$$

$$-1; z = \frac{\begin{vmatrix} 3 & -2 & -3 \\ 5 & 3 & 0 \\ 2 & 6 & 6 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & 4 \\ 5 & 3 & 1 \\ 2 & 6 & -5 \end{vmatrix}} = \frac{42}{-21} = -2.$$