FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS AND STATISTICS
FINAL EXAMINATION
MATH 221 (L05)

FALL 2006 Time: 3 hours

[10] 1. S olve the system:

$$\begin{array}{rcrcrcrcrcl} x & & & - & z & + & 2u & + & w & = & 2 \\ -2x & + & y & + & 2z & - & u & & & = & -7 \\ x & + & y & - & z & + & 3u & + & w & = & -1 \end{array}$$

[10] 2. L et $A = \begin{bmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{bmatrix}$.

(a) Find all values of x so that A is not invertible.

(b) Is it true that if A is not invertible then the system $AX = 0$ has no solutions? Explain.

[10] 3. L et $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

(a) Find an invertible matrix U such that $UA = B$.

(b) Express U^{-1} as a product of elementary matrices.

[10] 4. L et $A = \begin{bmatrix} 3 & -4 \\ 1 & -2 \end{bmatrix}$.

(a) Find an invertible matrix P and a diagonal matrix D so that $P^{-1}AP = D$.

(b) Compute A^7.

[10] 5. Le t $A^{-1} = \begin{bmatrix} 2 & 1 & 2 \\ -3 & -1 & -1 \\ 5 & 2 & 1 \end{bmatrix}$.

(a) Find $\det A$.

(b) Find $\det \left(A^{-1} + 2adj A\right)$.

[10] 6. L et A be a square matrix. Prove the following statements:

(a) If A is not invertible then 0 is an eigenvalue of A.

(b) If A is diagonalizable then A^T is also diagonalizable.

[10] 7. F or the following, express your answers in the form $a + bi$ where a and b are real numbers.

(a) Compute $\left(1 - \sqrt{3}i\right)^{10}$.

(b) Find all complex numbers z so that $z^4 = -16$.

[10] 8. Co nsider the points $A\,(2, 1, -2)$, $B\,(4, 1, 0)$ and $C\,(6, 3, 0)$.

(a) Find the internal angles of the triangle with vertices A, B and C.

(b) Find an equation of the plane containing the points A, B and C.

[10] 9. L et P_1 be the plane with equation $x + 2y - z = 2$ and P_2 be the plane with equation $2x - y + z = 2$. Let L be the line of intersection of the planes P_1 and P_2.

(a) Is the point $A(1, 1, 1)$ on both of the planes P_1 and P_2? Explain.

(b) Find an equation of the line L.

(c) Find the shortest distance between the point $B\,(4, -3, -3)$ and the line L, also find the point Q on the line L that is closest to B.

[10] 10. L et $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T\left[\begin{smallmatrix}2\\1\end{smallmatrix}\right] = \left[\begin{smallmatrix}1\\2\end{smallmatrix}\right]$ and $T\left[\begin{smallmatrix}3\\2\end{smallmatrix}\right] = \left[\begin{smallmatrix}2\\1\end{smallmatrix}\right]$.

(a) Find the matrix of T; that is, find a matrix A so that $T\overrightarrow{v} = A\overrightarrow{v}$ for all $\overrightarrow{v} \in \mathbb{R}^2$.

(b) Is T invertible? If T is invertible, find the matrix of T^{-1}.

(c) Is there a vector $\overrightarrow{a} \in \mathbb{R}^2$ so that $T\overrightarrow{a} = \left[\begin{smallmatrix}-3\\7\end{smallmatrix}\right]$? If so, find $\overrightarrow{a}$.

End of Examination