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Q#1(b) Find the roots of the complex quadratic equation.

$$x^2 - x + (1-i) = 0$$

solution:

Start by using the quadratic equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{where} \quad ax^2 + bx + c = 0.$$

In our case,

$$a = 1, \quad b = -1, \quad c = 1-i$$

Now sub in a, b, c and simplify:

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1-i)}}{2(1)} = \frac{1}{2}\left(1 \pm \sqrt{-3+4i}\right)$$

$$x = \frac{1}{2}(1 \pm w), \quad w = \sqrt{-3+4i}.$$

We want to eliminate the radical in w so that we can simplify the expression for x, that is, we want w in the form of $w = a+bi$.

So, square w and compare.

$$w = a+bi$$

$$w^2 = (a+bi)^2$$

$$= a^2 + 2abi + b^2(i)^2$$

$$= a^2 + 2abi - b^2$$

$$w^2 = (a^2-b^2) + (2ab)i$$

$w = \sqrt{-3+4i}$

$w^2 = (\sqrt{-3+4i})^2$

$w^2 = -3+4i$

Compare this with $w^2 = \overbrace{(a^2-b^2)}^{\text{real}} + \overbrace{(2ab)}^{\text{imaginary}} i$

We compare real and imaginary parts, so

$a^2 - b^2 = -3$ ... (1)

$2ab = 4$ ... (2)

From equation (2) $b = \frac{4}{2a} = \frac{2}{a}$. Sub this into

equation (1).

$a^2 - \left(\frac{2}{a}\right)^2 = -3 \Rightarrow a^2 - \frac{4}{a^2} = -3$

Now multiply through by a^2.

$a^2 \times \left(a^2 - \frac{4}{a^2} = -3\right) \Rightarrow a^4 - 4 = -3a^2$

$\Rightarrow a^4 + 3a^2 - 4 = 0$ (This is a quadratic in a^2.)

Factor: $(a^2+4)(a^2-1) = 0$.

$a^2 + 4 = 0 \Rightarrow a^2 = -4 \leftarrow$ (a must be real so we cannot use this one.)

$a^2 - 1 = 0 \Rightarrow a^2 = 1$

$a = \pm 1$

But we know $b = \frac{2}{a}$ so $b = \frac{2}{\pm 1} = \pm 2$.

$a = \pm 1,\ b = \pm 2$

Finally, since $w = a+bi$, then $w = \pm(1+2i)$. Now sub this into $x = \frac{1}{2}(1 \pm w)$, to get $x = \frac{1}{2}(1 \pm (1+2i)) = -i,\ 1+i$.