

Practice Problems S2

1. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 9 & 7 \\ -1 & 0 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 1 & 1 \\ -1 & 4 & 1 \end{bmatrix}$.

Compute AB , BA^T , BC and CB .

2. Let
$$\begin{cases} x_1 - 2x_2 + x_3 - 4x_4 = 1 \\ x_1 + 3x_2 + 7x_3 + 2x_4 = 2 \\ x_1 - 12x_2 - 11x_3 - 16x_4 = -1 \end{cases}.$$

- (a) Find basic solutions to the associated homogeneous system;
(b) Find a particular solution to the system.

3. Find basic solutions and particular solutions for the system $AX = B$, where

$$A = \begin{bmatrix} 2 & 1 & -1 & -1 \\ 3 & 1 & 1 & -2 \\ -1 & -1 & 2 & 1 \\ -2 & -1 & 0 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 \\ -2 \\ 2 \\ 3 \end{bmatrix}.$$

4. Find the inverses of the following matrices:

(a) $\begin{bmatrix} 7 & 4 \\ 3 & 2 \end{bmatrix}$; (b) $\begin{bmatrix} 1 & 3 & 2 \\ 1 & 1 & 1 \\ -1 & 4 & 1 \end{bmatrix}$.

5. Use matrix inversion to solve the following systems of linear equations:

(a) $\begin{cases} 7x + 4y = 2 \\ 3x + 2y = -2 \end{cases}$; (b) $\begin{cases} x + 3y + 2z = 5 \\ x + y + z = 1 \\ -x + 4y + z = 5 \end{cases}.$

6. Determine whether the following matrices are elementary matrices or not (explain your answer):

(a) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, (b) $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$, (c) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, (d) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

(e) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$, (f) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix}$.

7. Find an invertible matrix U such that the product UA has a reduced row-echelon form if

$$A = \begin{bmatrix} 1 & -1 & 3 & 5 \\ 3 & -2 & 1 & -2 \\ -1 & 1 & 1 & 3 \end{bmatrix}.$$

8. Express the following matrix as a product of elementary matrices:

$$A = \begin{bmatrix} 7 & 4 \\ 3 & 2 \end{bmatrix}.$$