

Practice Problems S1

1. Consider the following system of linear equations in x and y

$$\begin{cases} x + ay = 1 \\ ax + 4y = 2 \end{cases},$$

where $a \in \mathbb{R}$ is a parameter.

- (a) write down the coefficient matrix and the augmented matrix of the system;
- (b) Find all values of a and b (if any) for which the system has no solution, exactly one solution or infinitely many solutions.

2. Let

$$\begin{bmatrix} 1 & -1 & 3 & 5 \\ 3 & -2 & 1 & -2 \\ -1 & 1 & 1 & 3 \end{bmatrix}.$$

- (a) Carry the matrix A to a row-echelon form;
- (b) Find the rank of A ;
- (c) Use the row echelon form of A from part (a) to solve (back substitution) the system

$$\begin{cases} x_1 - x_2 + 3x_3 = 5 \\ 3x_1 - 2x_2 + x_3 = -2 \\ -x_1 + x_2 + x_3 = 3 \end{cases}.$$

3. (a) Find the reduced row-echelon form of the matrix

$$\begin{bmatrix} 1 & -1 & 3 & 1 & 3 \\ -1 & -2 & 6 & 2 & -6 \\ 2 & 1 & 3 & 5 & 3 \\ 2 & -2 & 12 & 8 & 0 \end{bmatrix}.$$

(b) Solve the system

$$\begin{cases} x_1 - x_2 + 3x_3 + x_4 = 3 \\ -x_1 - 2x_2 + 6x_3 + 2x_4 = -6 \\ 2x_1 + x_2 + 3x_3 + 5x_4 = 3 \\ 2x_1 - 2x_2 + 12x_3 + 8x_4 = 0 \end{cases} .$$

4. Solve the following system:

$$\begin{cases} x_1 - x_2 + 3x_3 + x_4 = 0 \\ -x_1 - 2x_2 + 6x_3 + 2x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + 5x_4 = 0 \\ 2x_1 - 2x_2 + 12x_3 + 8x_4 = 0 \end{cases} .$$

Recommended Problems:

Pages: 7 - 8: 1, 7, 8, 9, 10, 12

15 - 16: 1; 2(a); 3(a); 5(a), (b); 6(b); 7(a), (b); 8; 9(b); 11.