

## Practice Problems S3

1. Determine whether the following matrices are elementary matrices or not; write down the inverses of the elementary matrices (explain your answer):

$$(a) \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, (b) \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, (c) \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, (d) \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$(e) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}, (f) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix}.$$

2. Find an invertible matrix  $U$  such that the product  $R = UA$  is the reduced row-echelon form of  $A$  if

$$A = \begin{bmatrix} 1 & -1 & 3 & 5 \\ 3 & -2 & 1 & -2 \\ -1 & 1 & 1 & 3 \end{bmatrix}.$$

3. Express the following matrix as a product of elementary matrices:

$$A = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}.$$

4. Find the matrix of the reflection in the line  $y = -x$ .
5. Find a rotation or a reflection that is equal to
  - (a) reflection in the  $y$ -axis followed by rotation through  $\pi/2$ ;
  - (b) rotation through  $\pi/2$  followed by reflection in the line  $y = x$ .

6. Given  $T([1 \ -2]^T) = [3 \ 4]^T$  and  $T([-2 \ 5]^T) = [-1 \ 4]^T$ , find  $T([-4 \ 3]^T)$  if  $T$  is a linear transformation.
7. Consider a Markov chain that starts in state 1 with transition matrix  $P = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$ .
- (a) Explain why the chain is regular.
  - (b) Find the probability that the chain is in state 1 after 2 transitions.
  - (c) Find the steady-state vector for the chain.

**Recommended Problems:**

Pages 68 - 69: 1; 2a, b; 3a; 5a, b; 6 a,b; 7; 8b, c;

Pages 80-81: 1. b, c; 2. a; 3, 4, 5, 9, 10, 12; Pages 101-102: 1, 2, a, c;

Page 101-102: 1. a, b, f, g, h, k, l, m, n, o, p; 5. a, b; 6, 7, 8, 9, 11, 13, 14, 15;