1. Find all solutions to the linear system:
$$\begin{array}{rrrrrcr} x_1 & -2x_2 & & +2x_4 & +x_5 & = & 2 \\ -2x_1 & +4x_2 & +x_3 & -5x_4 & & = & -7 \\ x_1 & -2x_2 & +x_3 & +x_4 & +x_5 & = & -1 \end{array}.$$

   Solution: The reduction of the augmented matrix to reduced form is
$$\begin{bmatrix} 1 & -2 & 0 & +2 & 1 & 2 \\ -2 & +4 & 1 & -5 & 0 & -7 \\ 1 & -2 & 1 & 1 & 1 & -1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & -2 & 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & -1 & 0 & -3 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

   Hence the nonleading variables become parameters: $x_2 = s$, and $x_4 = t$. We solve for the leading variables in terms of the parameters: $x_1 = 2 + 2s - 2t$, $x_3 = -3 + t$ and $x_5 = 0$. Hence the general solution is $X = \begin{bmatrix} 2+2s-2t \\ s \\ -3+t \\ t \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -3 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$.

2. Suppose that $A^{-1} = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 1 & 1 \\ -1 & 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -4 \\ 0 & 5 \\ -7 & 0 \end{bmatrix}$. Find a matrix X such that $AX = B$.

   Solution: Multiply the equation $AX = B$ on the left by A^{-1} to get $A^{-1}AX = A^{-1}B$, that is $X = IX = A^{-1}B$. In this case $X = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 1 & 1 \\ -1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ 0 & 5 \\ -7 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -13 \\ -4 & -7 \\ -36 & 4 \end{bmatrix}$.

3. Find conditions on a, b and c such that the system $\begin{array}{rcrcrcl} x_1 & - & x_2 & + & 3x_3 & = & a \\ 2x_1 & & & + & x_3 & = & b \\ x_1 & + & 3x_2 & - & 7x_3 & = & c \end{array}$ has zero, one or infinitely many solutions.

   Solution: The reduction of the augmented matrix to reduced form leads to
$$\begin{bmatrix} 1 & -1 & 3 & a \\ 2 & 0 & 1 & b \\ 1 & 3 & -7 & c \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & -1 & 3 & a \\ 0 & 2 & -5 & b-2a \\ 0 & 4 & -10 & c-a \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & -1 & 3 & a \\ 0 & 2 & -5 & b-2a \\ 0 & 0 & 0 & c-(2b-3a) \end{bmatrix}$$

   If $c \neq 2b - 3a$ there is no solution; If $c = 2b - 3a$ the rank is $r = 2$ so the number of parameters is $n - r = 3 - 2 = 1$, and so there are infinitely many solutions. There is no choice of a, b and c that leads to a unique solution.

4. Find the matrix A if it satisfies the following equation:
$$7A + \begin{bmatrix} 4 & 3 & -1 \\ 2 & 7 & 0 \end{bmatrix}^T = \left\{2A^T - 3\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & 1 \end{bmatrix}\right\}^T.$$

   Solution: Performing the transpose on the right gives
$$7A + \begin{bmatrix} 4 & 3 & -1 \\ 2 & 7 & 0 \end{bmatrix}^T = (2A^T)^T - \left(3\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & 1 \end{bmatrix}\right)^T = 2A - 3\begin{bmatrix} 1 & 0 \\ 1 & -2 \\ 2 & 1 \end{bmatrix}.$$

   Subtracting $2A$ from both sides gives $5A = -3\begin{bmatrix} 1 & 0 \\ 1 & -2 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ 3 & 7 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -7 & -2 \\ -6 & -1 \\ -5 & -3 \end{bmatrix}$. Hence $A = \frac{1}{5}\begin{bmatrix} -7 & -2 \\ -6 & -1 \\ -5 & -3 \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} 7 & 2 \\ 6 & 1 \\ 5 & 3 \end{bmatrix}$.

5. Find the inverse of $A = \begin{bmatrix} 1 & -1 & -1 \\ 2 & -1 & -4 \\ 1 & -2 & 2 \end{bmatrix}$.

   Solution: We use the matrix inversion algorithm: $[A\ I] \to [I\ A^{-1}]$. Here this is:

$$\left[\begin{array}{rrrrrr} 1 & -1 & -1 & 1 & 0 & 0 \\ 2 & -1 & -4 & 0 & 1 & 0 \\ 1 & -2 & 2 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{rrrrrr} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & -1 & 3 & -1 & 0 & 1 \end{array}\right] \to \left[\begin{array}{rrrrrr} 1 & 0 & -3 & -1 & 1 & 0 \\ 0 & 1 & -2 & -2 & 1 & 0 \\ 0 & 0 & 1 & -3 & 1 & 1 \end{array}\right] \to \left[\begin{array}{rrrrrr} 1 & 0 & 0 & -10 & 4 & 3 \\ 0 & 1 & 0 & -8 & 3 & 2 \\ 0 & 0 & 1 & -3 & 1 & 1 \end{array}\right]$$

Hence $A^{-1} = \left[\begin{array}{rrr} -10 & 4 & 3 \\ -8 & 3 & 2 \\ -3 & 1 & 1 \end{array}\right]$.

6. Let A and B denote invertible square matrices of the same size. If $(AB)^2 = A^2B^2$, show that $AB = BA$.

   Solution: The condition is $ABAB = AABB$. Since A^{-1} exists, we left multiply by it to get $A^{-1}ABAB = A^{-1}AABB$, that is $IBAB = IABB$, that is $BAB = ABB$. Now right multiply by B^{-1} to get $BABB^{-1} = ABBB^{-1}$, that is $BAI = ABI$, that is $BA = AB$.

7. Find A if $\left\{4A^{-1} + \left[\begin{smallmatrix} 2 & -2 \\ 1 & 0 \end{smallmatrix}\right]^T\right\}^{-1} = 5A$.

   Solution: Inverting both sides gives $4A^{-1} + \left[\begin{smallmatrix} 2 & 1 \\ -2 & 0 \end{smallmatrix}\right] = \frac{1}{5}A^{-1}$. Subtracting $\frac{1}{5}A^{-1}$ from both sides gives $\frac{19}{5}A^{-1} = -\left[\begin{smallmatrix} 2 & 1 \\ -2 & 0 \end{smallmatrix}\right] = \left[\begin{smallmatrix} -2 & -1 \\ 2 & 0 \end{smallmatrix}\right]$, so $A^{-1} = \frac{5}{19}\left[\begin{smallmatrix} -2 & -1 \\ 2 & 0 \end{smallmatrix}\right]$. Finally, inverting both sides gives $A = \frac{19}{5}\left[\begin{smallmatrix} 0 & 1 \\ -2 & -2 \end{smallmatrix}\right]^{-1} = \frac{19}{5}\left(\frac{1}{2}\left[\begin{smallmatrix} 0 & 1 \\ -2 & -2 \end{smallmatrix}\right]\right) = \frac{19}{10}\left[\begin{smallmatrix} 0 & 1 \\ -2 & -2 \end{smallmatrix}\right]$.

8. Let A and B be square matrices of the same size. If A, B and AB are all symmetric, show that $AB = BA$.

   Solution: We are given $A^T = A$, $B^T = B$ and $(AB)^T = AB$. But $(AB)^T = B^TA^T$ always holds, so we have $AB = (AB)^T = B^TA^T = BA$, as required.

9. Let A and B be 3×3 matrices with A invertible. If $\det(A^{-1}B^T) = 2$ and $\det(2AB^2) = \frac{1}{2}$, find $\det A$ and $\det B$.

   Solution: Write $\det A = a$ and $\det B = b$. Then $2 = \det(A^{-1}B^T) = \det(A^{-1})\det(B^T) = \frac{1}{a}b$, so $b = 2a$. Similarly $\frac{1}{2} = 2^3 \det A\, (\det B)^2 = 8ab^2$. Hence $\frac{1}{16} = ab^2 = a(2a)^2 = 2a^3$, so $a^3 = \frac{1}{64}$. This gives $a = \frac{1}{4}$, whence $b = 2a = \frac{1}{2}$.

10. Let $A = \left[\begin{smallmatrix} 2 & 3 \\ 4 & 1 \end{smallmatrix}\right]$. Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

    Solution: The characteristic polynomial is

$$c_A(x) = \det(xI - A) = \det\left[\begin{array}{cc} x-2 & -3 \\ -4 & x-1 \end{array}\right] = x^2 - 3x - 10 = (x-5)(x+2).$$

    So the eigenvalues of A are $\lambda_1 = 5$ and $\lambda_2 = -2$. The eigenvectors corresponding to any eigenvalue λ are the nonzero solutions X to $(\lambda I - A)X = 0$. Hence:

    If $\lambda_1 = 5$ then $\lambda_1 I - A = \left[\begin{smallmatrix} 3 & -3 \\ -4 & 4 \end{smallmatrix}\right]$ so the eigenvectors are $X = t\left[\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}\right]$, $t \neq 0$. Take $X_1 = \left[\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}\right]$.

    If $\lambda_2 = -2$, then $\lambda_2 I - A = \left[\begin{smallmatrix} -4 & -3 \\ -4 & -3 \end{smallmatrix}\right]$ so the eigenvectors are $X = t\left[\begin{smallmatrix} -3 \\ 4 \end{smallmatrix}\right]$, $t \neq 0$. Take $X_1 = \left[\begin{smallmatrix} -3 \\ 4 \end{smallmatrix}\right]$.

    The matrix $P = [X_1\ X_2] = \left[\begin{smallmatrix} 1 & -3 \\ 1 & 4 \end{smallmatrix}\right]$ is invertible, so $P^{-1}AP = \left[\begin{smallmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{smallmatrix}\right] = \left[\begin{smallmatrix} 5 & 0 \\ 0 & -2 \end{smallmatrix}\right]$ is diagonal.

11. Let $A = \left[\begin{smallmatrix} 1 & c & 2 \\ c & 1 & c \\ 0 & c & 1 \end{smallmatrix}\right]$. Determine all the values of c for which A is invertible. Justify your answer.

    Solution: We use the fact that a matrix is invertible if and only if its determinant is nonzero. Here:

$$\det A = \det\left[\begin{array}{ccc} 1 & c & 2 \\ c & 1 & c \\ 0 & c & 1 \end{array}\right] = \det\left[\begin{array}{ccc} 1 & c & 2 \\ 0 & 1-c^2 & -c \\ 0 & c & 1 \end{array}\right] = \det\left[\begin{array}{cc} 1-c^2 & -c \\ c & 1 \end{array}\right] = (1-c^2) + c^2 = 1.$$

    Hence A is invertible for *any* value of c.

12. Let $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$. Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

Solution: The characteristic polynomial is

$$c_A(x) = \det(xI - A) = \det\begin{bmatrix} x+1 & -4 \\ -2 & x-1 \end{bmatrix} = x^2 - 9 = (x-3)(x+3).$$

So the eigenvalues of A are $\lambda_1 = 3$ and $\lambda_2 = -3$. The eigenvectors corresponding to any eigenvalue λ are the nonzero solutions X to $(\lambda I - A)X = 0$. Hence:

If $\lambda_1 = 3$ then $\lambda_1 I - A = \begin{bmatrix} 4 & -4 \\ -2 & 2 \end{bmatrix}$ so the eigenvectors are $X = t\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $t \neq 0$. Take $X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

If $\lambda_2 = -3$, then $\lambda_2 I - A = \begin{bmatrix} -2 & -4 \\ -2 & -4 \end{bmatrix}$ so the eigenvectors are $X = t\begin{bmatrix} -4 \\ 2 \end{bmatrix}$, $t \neq 0$. Take $X_2 = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$.

The matrix $P = [X_1\ X_2] = \begin{bmatrix} 1 & -4 \\ 1 & 2 \end{bmatrix}$ is invertible, so $P^{-1}AP = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & -3 \end{bmatrix}$ is diagonal.

13. Let A denote a diagonalizable $n \times n$ matrix. If every eigenvalue λ of A satisfies $\lambda^2 = 1$, show that $A^2 = I$.

Solution: Since A is diagonalizable, let $P^{-1}AP = D = \operatorname{diag}(\lambda_1, \lambda_2, \cdots, \lambda_n)$. These λ_i are eigenvalues of A so $D^2 = \operatorname{diag}(\lambda_1^2, \lambda_2^2, \cdots, \lambda_n^2) = \operatorname{diag}(1, 1, \cdots, 1) = I$ by assumption. But then solving $P^{-1}AP = D$ for A yields $A = PDP^{-1}$, so $A^2 = PD^2P^{-1} = PIP^{-1} = I$, as reqired.

14. Determine whether or not the matrix $A = \begin{bmatrix} -1 & 1 & -1 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{bmatrix}$ is diagonalizable. Support your answer.

Solution: We have:

$$c_A(x) = \det(xI - A) = \det\begin{bmatrix} x+1 & -1 & 1 \\ 0 & x-1 & 0 \\ -2 & -1 & x-2 \end{bmatrix} = (x-1)\det\begin{bmatrix} x+1 & 1 \\ -2 & x-2 \end{bmatrix} = x(x-1)^2.$$

If $\lambda_1 = 1$ the eigenvectors are the nonzero solutions of $(\lambda_1 I - A)X = 0$. Here

$$\lambda_1 I - A = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 0 & 0 \\ -2 & -1 & -1 \end{bmatrix} \to \begin{bmatrix} 2 & -1 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Hence the system $(\lambda_1 I - A)X = 0$ has a one parameter family of solutions so, since λ_1 has multiplicity 2, the matrix A is *not* diagonalizable by Theorem 4 §3.3.

15. If A is an $n \times n$ diagonalizable matrix such that $A^2 = 0$, show that $A = 0$. [Hint: Show first that $\lambda = 0$ for every eigenvalue λ of A.]

Solution: If λ is an eigenvalue of A, then $AX = \lambda X$ for some column $X \neq 0$. Multiply by A to get $A^2X = \lambda(AX)$; that is $0X = \lambda(\lambda X)$; that is $\lambda^2 X = 0$. Since $X \neq 0$ this shows that $\lambda^2 = 0$, whence $\lambda = 0$.

Now, since A is diagonalizable, we have $P^{-1}AP = D = \operatorname{diag}(\lambda_1, \lambda_2, \cdots, \lambda_n)$ where the λ_i are the eigenvalues of A. By the above, this shows that $P^{-1}AP = 0$, so $A = P0P^{-1} = 0$.

16. Determine whether or not the matrix $A = \begin{bmatrix} 1 & 0 & -4 \\ 1 & 3 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ is diagonalizable. Justify your answer.

Solution: We have:

$$c_A(x) = \det(xI - A) = \det\begin{bmatrix} x-1 & 0 & 4 \\ -1 & x-3 & -2 \\ 1 & 0 & x-1 \end{bmatrix} = (x-3)\det\begin{bmatrix} x-1 & 4 \\ 1 & x-1 \end{bmatrix} = (x-3)^2(x+1).$$

If $\lambda_1 = 3$ the eigenvectors are the nonzero solutions of $(\lambda_1 I - A)X = 0$. Here

$$\lambda_1 I - A = \begin{bmatrix} 2 & 0 & 4 \\ -1 & 0 & -2 \\ 1 & 0 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Hence the system $(\lambda_1 I - A)X = 0$ has a two parameter family of solutions so, since λ_1 has multiplicity 2, the matrix A *is* diagonalizable by Theorem 4 §3.3.

17. Find the point Q on the line $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$ that is closest to the point $P(1, 0, -2)$.

Solution: Write $P_0(2, -1, 3)$—a point on the line, and write $\vec{v} = \overrightarrow{P_0P} = \begin{bmatrix} -1 \\ 1 \\ -5 \end{bmatrix}$. If $\vec{d} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$ is the direction vector of the line, we have $\overrightarrow{P_0Q} = \text{proj}_{\vec{d}}(\vec{v}) = \frac{\vec{v} \bullet \vec{d}}{\|\vec{d}\|^2} \vec{d} = \frac{-16}{10} \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} = -\frac{8}{5} \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$. If $\vec{p}_0$ and $\vec{q}$ denote the position vectors of P_0 and Q respectively, we have $\vec{q} = \vec{p}_o + \overrightarrow{P_0Q} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} - \frac{8}{5} \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} \\ -1 \\ -\frac{9}{5} \end{bmatrix}$, so $Q = Q(\frac{2}{5}, -1, -\frac{9}{5})$.

Note, we can check our arithmetic here: The vector $\vec{v} -$ proj$_{\vec{d}}(\vec{v})$ should be orthogonal to $\vec{d}$; that is $\begin{bmatrix} -1 \\ 1 \\ -5 \end{bmatrix} + \frac{8}{5} \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}$ should be orthogonal to $\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$; and this is true as the dot product is zero.

18. Let P and Q be points where $P = P(1, -2, 0)$, and let $\vec{v} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$. If $\overrightarrow{PQ}$ is parallel to $\vec{v}$ and $\left\|\overrightarrow{PQ}\right\| = 6$, find the coordinates of the point Q. [There are two such points Q.]

Solution: Let $\vec{p}$ and $\vec{q}$ denote the position vectors of P and Q respectively, so that $\vec{p} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$. We have $\overrightarrow{PQ} = t\,\vec{v}$ for some scalar t, so $6 = \left\|\overrightarrow{PQ}\right\| = |t|\,\|\vec{v}\| = 3\,|t|$. Hence $|t| = 2$, so $t = \pm 2$. This means $\overrightarrow{PQ} = \pm 2\vec{v}$, so we obtain $\vec{q} = \vec{p} + \overrightarrow{PQ} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} \pm 2 \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$. Hence the two points Q are $Q(3, -6, 4)$ and $Q(-1, 2, -4)$.

19. Find the point Q on the plane with equation $2x - y + 3z = 1$ that is closest to the point $P(2, 1, -3)$.

Solution: Choose a point $P_0(0, -1, 0)$ in the plane, and read off a normal $\vec{n} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ to the plane. Write $\vec{v} = \overrightarrow{P_0P} = \begin{bmatrix} 2 \\ 2 \\ -3 \end{bmatrix}$, and compute $\vec{v}_1 = \text{proj}_{\vec{n}}(\vec{v}) = \frac{\vec{v} \bullet n}{\|\vec{n}\|^2} \vec{n} = \frac{-7}{14} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = -\frac{1}{2} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$. If $\vec{p}$ and $\vec{q}$ denote the position vectors of P and Q respectively, then $\vec{q} = \vec{p} -$ proj$_{\vec{n}}(\vec{v}) = \frac{1}{2} \begin{bmatrix} 6 \\ 1 \\ -3 \end{bmatrix}$. Hence $Q = Q(3, \frac{1}{2}, -\frac{3}{2})$.

Note that the work can be checked: The vector $\vec{v} -$ proj$_{\vec{n}}(\vec{v}) = \frac{3}{2} \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ should be orthogonal to $\vec{n} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$, and this is indeed the case as the dot product is zero.

20. (a) Find the angle θ between $\vec{v} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}$.

(b) Consider the points $A(2, -1, 1)$, $B(1, 0, 0)$ and $C(2, 2, 1)$. Determine if the triangle with these points as vertices is a right-angled triangle. Justify your answer.

Solution: (a). $\cos\theta = \frac{\vec{v} \bullet \vec{w}}{\|\vec{v}\|\|\vec{w}\|} = \frac{6+6-3}{\sqrt{63}\sqrt{6}} = \frac{9}{18} = \frac{1}{2}$. Hence $\theta = \frac{\pi}{3}$.

(b) We have $\overrightarrow{AB} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$, $\overrightarrow{AC} = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$ and $\overrightarrow{BC} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$. Thus $\overrightarrow{AB} \bullet \overrightarrow{BC} = 0$, so the angle at vertex B is a right angle.