



UNIVERSITY OF CALGARY

Faculty of Science
Department of Mathematics & Statistics

MIDTERM #1 - MATH 221 - L18 October 8, 2004

Your family name: _____

Your first name: _____

Your signature: _____

Your student number: _____

INSTRUCTIONS:

- I. Fill out the above information BEFORE starting this exam.
- II. **Show all your work**, use the back of the previous page for rough work and clearly insert the main steps and answers in the provided space.
- III. Calculators allowed, but no other material.
- IV. There are 3 questions and 4 pages to this exam.
- V. Time allowed is 50 minutes.

PROBLEM	#1	#2	#3	TOTAL
MARKS	/6	/8	/6	/20

Question 1 (6 points)

Consider the matrix equation

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \mathbf{X} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

[1] a) What must the size of the matrix X be? ANSWER:_____

[2] b) Transform the above equation into the more convenient form $AX = B$, and give both matrices A and B .

[1] c) With justification, how many solutions X of the equation will there be?

[2] d) Find all solutions X of the equation, if any.

Question 2 (8 points)

Consider a system of linear equations in the variables x, y, z of the form

$$\begin{aligned}x & \quad + 3z = 0 \\x & + y + 4z = 0 \\x & + ay + 5z = 0\end{aligned}$$

where a can be any number.

[3] a) Find the general solution of the system when $a = 2$.

[3] b) With full explanation, find all values of a such that the system has infinitely many solutions.

[1] c) Find all values of a such that the system has a unique solution.

[1] d) Find all values of a such that the system has a no solution.

Question 3 (6 points)

(Fictional) Data shows that from one year to the next, 5% of Canada's highly educated and motivated people (*hemp*) migrate to the US, and conversely 1% of the US' *hemp* to Canada.

[2] a) Assuming for this problem that at any time no such *hemp* moves out of these two countries entirely, or comes in from anywhere else, express the above population movement as a Markov chain. In particular define the state vectors, and provide the transition matrix.

[1] b) Assuming that this year Canada has 10 million *hemp*, and the US 100 million such people, find how many *hemp* will reside in each country next year.

[2] c) Find the steady-state vector for the Markov chain, and also the eventual *hemp* population of each country if this trend continues.

[1] d) From the above results, compute how many *hemp* we are to lose/gain in the long run, and briefly argue whether Canada's brain drain is in a crisis.