UNIVERSITY OF CALGARY

Faculty of Science
Department of Mathematics & Statistics

MIDTERM #1 - MATH 221 - L11

October 6, 2006

Your family name: ______________________________

Your first name: ______________________________

Your signature: ______________________________

Your student number: ______________________________

INSTRUCTIONS:
I. Fill out the above information BEFORE starting this exam.
II. **Show all your work**, use the back of the previous page for rough work and clearly insert the main steps and answers in the provided space.
III. Calculators are not allowed, and no other material.
IV. There are 3 questions and 4 pages to this exam.
V. Time allowed is 50 minutes.

PROBLEM	#1	#2	#3	TOTAL
MARKS	/6	/8	/6	/20

Question 1 (6 points)
Answer the following questions by TRUE or FALSE, and provide a short but precise explanation supporting your claim:

[2] a) If A is an $m \times n$ matrix, and $m < n$, then the homogeneous system $AX = 0$ has infinitely many solutions.

ANSWER:TRUE

EXPLANATION:
We have $rankA \leq m < n$, so there are $n - rankA > 0$ parameters and therefore infinitely many solutions.

[2] b) If a system of linear equations is of the form $AX = B$ has a solution for some B, then the system has a solution for any B.

ANSWER:FALSE

EXPLANATION:
To show that a statement is true, you need to justify (prove) it. But to show one is false, you only need to provide a counterxample.
One is obtained by taking $A = [0]$. There is a solution with $B = [0]$, but not with $B = [1]$.

[2] c) If a system of linear equations is of the form $AX = B$ where A is **not** invertible, then the system has no solution.

ANSWER:FALSE

EXPLANATION:
Again you can take the system $[0]X = [0]$.

Question 2 (8 points)
Consider a system of linear equations in the variables x, y, z of the form

$$\begin{array}{lrrcl} x & +2y & +3z & = & 4 \\ x & +3y & +5z & = & 3 \\ x & +(a+2)y & +7z & = & 6 \end{array}$$

where a can be any number.

[2] a) Find the general solution of the system when $a = 2$.
With $a = 2$, we get an augmented matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 5 & 3 \\ 1 & 4 & 7 & 6 \end{bmatrix}$$

which has a ref

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and therefore no solution.
[2] b) With explanation, find all values of a such that the system has no solution.
With a, we get an augmented matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 5 & 3 \\ 1 & (a+2) & 7 & 6 \end{bmatrix}$$

and doing the corresponding row operations one gets to

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 4-2a & 3 \end{bmatrix}.$$

So if there is no solution, we must have $4 - 2a = 0$, and therefroe $a = 2$. So tat's the only possibility and we showed in a) that it does give no solution, so that's it!
[2] c) Find all values of a such that the system has a unique solution.
If $4-2a \neq 0$ (i.e. $a \neq 2$), then we can see form part b) (by dividing Row 3 by $4-2a \neq 0$ to yield rank 3) that there will be a unique solution.
[2] d) Find all values of a such that the system has infinitely many solutions.
We have already covered all values of a ($a = 2$ yields no sokution, $a \neq 2$ yields a unique one), so infinitely many solutions will never happen.

Question 3 (6 points)

[2] a) Define what it means for a matrix B to be an inverse of another matrix A.

$$AB = I = BA$$

[2] b) Does every nonzero matrix have an inverse? Explain.
No, and again we need a counterxample, and it must ne nonzero.
Let's take $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.
Then for any $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $AB = \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} \neq I$.

[2] c) How many inverses can a matrix A have? Explain.
Only one. If we had two such inverses, say $AB = I = BA$ and $AC = I = CA$, then

$$B = BI = B(AC) = (BA)C = IC = C.$$