



UNIVERSITY OF CALGARY

Faculty of Science
Department of Mathematics & Statistics

MIDTERM #2 - MATH 221 - L11 November 10, 2006

Your family name: _____

Your first name: _____

Your signature: _____

Your student number: _____

INSTRUCTIONS:

- I. Fill out the above information BEFORE starting this exam.
- II. **Show all your work**, use the back of the previous page for rough work and clearly insert the main steps and answers in the provided space.
- III. Calculators are not allowed, and no other material.
- IV. There are 3 questions and 4 pages to this exam.
- V. Time allowed is 50 minutes.

PROBLEM	#1	#2	#3	TOTAL
MARKS	/5	/6	/9	/20

Question 1 (5 points)

[1] a) Explain what it means that a transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is linear.

[1] b) Explain what it means that a transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a matrix transformation.

[2] c) If $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is linear, show that T is a matrix transformation and how to find the corresponding matrix A .

[1] d) Find the matrix of $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, if T is the reflection in the line $y = -x$.

Question 2 (6 points)

[1] a) Describe what is a steady-state S of a Markov Chain $S_{m+1} = PS_m$.

[1] b) What must be one of the eigenvalues of P if there is a steady-state S of the Markov Chain $S_{m+1} = PS_m$? Explain.

[3] c) Find all values of a such that $\lambda = 1$ is an eigenvalue of the following matrix

$$P = \begin{bmatrix} .8 & a \\ .2 & 1 - a \end{bmatrix}.$$

[1] d) Use your result in part c) to find all values of a such that the Markov Chain $S_{m+1} = PS_m$ has a steady-state where P is as in part c).

Question 3 (9 points)

Consider the number of ways x_k to fill a row parking lot with k spaces with Cars taking one space, and Minivans and SUVs each taking 2 spaces.

[2] a) Compute the values of x_k for small $k = 0, 1, 2, 3$.

[2] b) Rephrase the question using the technique of dynamical systems.

[5] c) Describe how you would compute x_k for large values of k with as much detail as possible.