MATHEMATICS 251 FINAL EXAMINATION, FALL 2001
TIME: 2 HOURS

SHOW ALL WORK. SIMPLIFY ALL ANSWERS AS MUCH AS POSSIBLE. NO CALCULATORS PLEASE. THE MARKS FOR EACH PROBLEM ARE GIVEN TO THE LEFT OF THE PROBLEM NUMBER. TOTAL MARKS [80].

[5] 1. Find and simplify $\dfrac{d}{dx}\left(\sqrt{x\tan^2 x - 4}\right)$.

[5] 2. Find and simplify $\dfrac{d}{dx}\left(\dfrac{\sin(6 - x^{3/2})}{e^{x^5} - 2}\right)$.

[6] 3. Find and simplify the equation of the tangent line to the curve $y = 2x - \cos 3x$ at the point where $x = 0$.

[6] 4. Find and simplify $\dfrac{dy}{dx}$ where $y \ln x - x \ln y = 8$.

[6] 5. USE THE DEFINITION OF DERIVATIVE to find $\dfrac{d}{dx}\left(\dfrac{1}{2x-1}\right)$.

[6] 6. Use the derivative of the natural logarithm function, and implicit differentiation, to prove the formula for $\dfrac{d}{dx}e^x$.

[8] 7. A spotlight is on the ground pointing towards a high wall 20 metres away. A child 1 metre tall is standing near the wall so that her shadow is cast onto the wall as in the picture. The child begins to run at 2 metres per second towards the spotlight. How fast is the length of her shadow changing at the instant that she is halfway between the wall and the spotlight?

[10] 8. For the function $f(x) = x - 3x^{2/3}$, find the following, if any: the x and y intercepts, horizontal and vertical asymptotes, intervals of increase and decrease, relative maxima and minima, intervals of concave up and concave down, and inflection points. Then draw the graph of the function.

[8] 9. Find the point(s) on the curve $x^2 - 2y^2 = 1$ which are closest to the point $(0, 6)$.

[5] 10. Find and simplify $\displaystyle\int \sec 2x \tan 2x \, dx$.

[5] 11. Find and simplify $\displaystyle\int_0^1 \frac{x^3}{(x^4+3)^{3/2}} \, dx$.

[5] 12. Find constants a and b so that the function $f(x) = \begin{cases} ax^2 + b, & x \le 2 \\ x^3, & x > 2 \end{cases}$ is differentiable at $x = 2$.

[5] 13. Find and simplify $\lim_{x\to 0}(e^{3x} - 1)\cot 4x$.

[80]

MATHEMATICS 249 FINAL EXAMINATION, FALL 2002
TIME: 2 HOURS

SHOW ALL WORK. SIMPLIFY ALL ANSWERS AS MUCH AS POSSIBLE. NO CALCULATORS PLEASE. THE MARKS FOR EACH PROBLEM ARE GIVEN TO THE LEFT OF THE PROBLEM NUMBER. TOTAL MARKS [75].

[5] 1. Find and simplify $\lim_{x\to -5}\left(\dfrac{25-x^2}{2x^2+3x-35}\right)$.

[5] 2. Find and simplify $\dfrac{d}{dx}\ln(x^3-\cos x)$.

[5] 3. Find and simplify $\dfrac{d}{dx}\Big(x\tan(e^{5x})\Big)$.

[5] 4. Find and simplify $\dfrac{d}{dx}\left(\dfrac{(1-x)^{1/3}}{3-\sqrt{x}}\right)$.

[6] 5. Do **ONE** of the following two problems.

(a) Use implicit differentiation to find and simplify $\dfrac{dy}{dx}$ where $\sin(5x-y)+2x=y^2+7$.

(b) Use logarithmic differentiation to find and simplify $\dfrac{d}{dx}\Big((\sec x)^{4x}\Big)$.

[6] 6. Find and simplify the equation of the tangent line to the curve $y=\dfrac{8}{(x-1)^2}$ at the point where $x=3$.

[6] 7. USE THE DEFINITION OF DERIVATIVE to find $\dfrac{d}{dx}\Big(\sqrt{4-3x}\Big)$.

[10] 8. You are given the function $f(x)=x-4\sqrt{x}$, and that

$$f'(x)=1-\frac{2}{\sqrt{x}}\ , \qquad f''(x)=\frac{1}{x^{3/2}}\ .$$

For the function $f(x)$, find: (a) the domain of f; (b) the critical points; (c) the intervals of increase and decrease; (d) the intervals of concave up and concave down; (e) all local maxima and local minima. Also find (f) the absolute maximum and absolute minimum of $f(x)$ on the interval $[0,25]$.

[6] 9. Find constants k and ℓ so that the function

$$f(x)=\begin{cases} k-2x & \text{if } x\le 2,\\ \ell\sqrt{x+7} & \text{if } x>2\end{cases}$$

is both continuous and differentiable at $x=2$.

[5] 10. Find and simplify $\int \frac{x^2}{(x^3-8)^3}\,dx$.

[5] 11. Find and simplify $\int e^{-x}\cos(e^{-x})\,dx$.

[5] 12. Find and simplify $\int_{-2}^{3}(5-4x)\,dx$.

[6] 13. A rectangle has its base on the x axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest **perimeter** the rectangle can have, and what are its dimensions?

$\overline{[75]}$

MATHEMATICS 249 FINAL EXAMINATION, WINTER 2003
TIME: 2 HOURS

SHOW ALL WORK. SIMPLIFY ALL ANSWERS AS MUCH AS POSSIBLE. NO CALCULATORS PLEASE. THE MARKS FOR EACH PROBLEM ARE GIVEN TO THE LEFT OF THE PROBLEM NUMBER. TOTAL MARKS [75].

[5] 1. Find and simplify $\dfrac{d}{dx}\left(\dfrac{x}{x^3 - \ln x}\right)$.

[5] 2. Find and simplify $\dfrac{d}{dx}\left(e^{\sec^2 x}\right)$.

[5] 3. Find and simplify $\dfrac{d}{dx}\left((\tan 3x)\sqrt{1-x^2}\right)$.

[5] 4. Use logarithmic differentiation to find and simplify $\dfrac{d}{dx}\left((x+1)^{\sqrt{x}}\right)$.

[8] 5. (a) Use implicit differentiation to find and simplify $\dfrac{dy}{dx}$ where $e^x + \ln y = \cos x + \sin(\pi y)$.

(b) Verify that the point $(0, 1)$ lies on the graph of $e^x + \ln y = \cos x + \sin(\pi y)$.

(c) Find the equation of the tangent line to the graph of $e^x + \ln y = \cos x + \sin(\pi y)$ at the point $(0, 1)$.

[5] 6. Prove the formula for $\dfrac{d}{dx}(\cot x)$. You may use the formulas for the derivatives of $\sin x$ and $\cos x$.

[6] 7. USE THE DEFINITION OF DERIVATIVE to find $\dfrac{d}{dx}\left(\dfrac{1}{4-x^2}\right)$.

[10] 8. For the function $f(x) = \dfrac{x^{1/3}}{x-2}$,

(a) show that $f'(x) = -\dfrac{2x+2}{3x^{2/3}(x-2)^2}$.

Then find (b) the domain of $f(x)$; (c) the critical points; (d) the intervals of increase and decrease; (e) all local maxima and local minima.

[5] 9. For the function $f(x) = xe^x$, you are given that $f'(x) = (x+1)e^x$ and $f''(x) = (x+2)e^x$. Find the intervals on which $f(x)$ is concave up and where it is concave down. Then find all points of inflection.

[5] 10. Find all constants k so that the function

$$f(x) = \begin{cases} x^3 + 4x^2 + x + 7 & \text{if } x \le k, \\ 7 - 3x & \text{if } x > k \end{cases}$$

is continuous at $x = k$. Also, for each such value of k, determine whether $f(x)$ is differentiable at $x = k$.

[5] 11. Find and simplify $\int \sqrt{\sin 4x}\cos 4x \; dx$.

[5] 12. Find and simplify $\int_0^1 (x^{1/3} - x^3) \; dx$.

[6] 13. Do **ONE** of the following two problems.

(a) A ladder 4 metres long is leaning against a house (see Figure (a)). Its base starts to slide away from the house at 1/2 metre per second. How fast is the top of the ladder moving down the house when the top is 3 metres from the ground?

(b) A rectangle has its base on the x axis, its left side on the y axis, and its upper right-hand vertex on the curve $y = 4/x^2$. (See Figure (b).) What is the smallest **perimeter** the rectangle can have, and what are its dimensions?

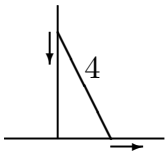

Figure (a)

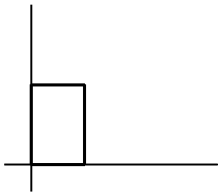

Figure (b)

[75]