MATHEMATICS 249 MIDTERM Fall 2000

SHOW ALL WORK. Marks for each problem are to the left of the problem number.
NO CALCULATORS PLEASE.

[5] 1. Find $\lim_{x\to 1/2}\left(\dfrac{5-10x}{2x^2-7x+3}\right)$.

[5] 2. Find $\lim_{x\to 0}\left(\dfrac{\sin(x^2-2x)}{x}\right)$.

[5] 3. Find $\dfrac{d}{dx}\left(\dfrac{\tan 2x}{x+8}\right)$.

[5] 4. Find $\dfrac{d}{dx}\sqrt{\sin x - x\cos x}$.

[5] 5. Find $\dfrac{d}{dx}\Big((7-\sec^7 x)^{-7}\Big)$.

[5] 6. USE THE DEFINITION OF DERIVATIVE to find $\dfrac{d}{dx}\sqrt{3-5x}$.

[5] 7. An object is moving on a straight line. Its position (distance from a fixed point) at any time t is given by the function $f(t) = 2t^2 - 5t + 2$. Find the instantaneous velocity of the object at time $t = 3$. (Use any method.)

[5] 8. Suppose that $f(x)$ and $g(x)$ are differentiable functions. Use the definition of derivative to prove that $\dfrac{d}{dx}(f(x)-g(x)) = \dfrac{d}{dx}f(x) - \dfrac{d}{dx}g(x)$.

[40]

MATHEMATICS 251 MIDTERM Fall 2001

SHOW ALL WORK. Marks for each problem are to the left of the problem number.
NO CALCULATORS PLEASE.

[5] 1. Find $\lim_{x\to 2}\left(\dfrac{2x^2+x-10}{8-2x^2}\right)$. (Do not use l'Hôpital's Rule.)

[5] 2. Find $\lim_{x\to 0}\left(\dfrac{x^2}{\sin^2 4x}\right)$. (Do not use l'Hôpital's Rule.)

[5] 3. Find and simplify $\dfrac{d}{dx}(x^2\sqrt{2-x^2})$.

[5] 4. Find and simplify $f'(x)$ where $f(x)=\sec(\tan x)-\sec x\tan x$.

[5] 5. Find and simplify $\dfrac{d}{dx}\left(\dfrac{4-\cos 3x}{3x^2+\sec 4x}\right)$.

[5] 6. Find and simplify $\dfrac{d}{dx}\sqrt{1-x\sin 2x}$.

[5] 7. USE THE DEFINITION OF DERIVATIVE to find $\dfrac{d}{dx}\sqrt{x^2-7}$.

[5] 8. Show that the function $f(x)=\begin{cases} x^3-8x, & x<2 \\ x^2-12, & x\geq 2 \end{cases}$ is continuous at $x=2$.

[5] 9. Find the equation of the tangent line to the graph of $y=(2x-1)^{-2}$ at the point where $x=1$.

[5] 10. Find the derivative of $\csc x$. You may use formulas for the derivatives of any of the other five trigonometric functions.

[50]

MATHEMATICS 249 MIDTERM Fall 2002

SHOW ALL WORK. Marks for each problem are to the left of the problem number.
NO CALCULATORS PLEASE.

[4] 1. Find $\lim_{x\to\infty}\left(\dfrac{4+x^2}{1+4x^2}\right)$.

[4] 2. Find $\lim_{x\to 6^+}\left(\dfrac{x-8}{x-6}\right)$.

[5] 3. Find and simplify $\lim_{x\to 1}\left(\dfrac{2x-\sqrt{5-x}}{x-1}\right)$.

[5] 4. Find and simplify $\dfrac{d}{dx}\left(\dfrac{\sqrt{x}}{x+\cos x}\right)$.

[5] 5. Find and simplify $\dfrac{d}{dx}\left(\sin^2(2x^2-x)\right)$.

[5] 6. Find and simplify $\dfrac{d}{dx}\left((14x-\tan 3x)^{5/2}\right)$.

[5] 7. USE THE DEFINITION OF DERIVATIVE to find $\dfrac{d}{dx}\left(\dfrac{1}{1-x}\right)$.

[6] 8. Find the equation of the tangent line to the graph of $y = 12x - 5x^3$ at the point where $x = 1$.

[5] 9. Use implicit differentiation to find and simplify dy/dx where $x^3 + y^2 = 5xy + 8$.

[6] 10. An object moves along a straight line so that its position (in metres) at any time t (in seconds) is given by the function $p(t) = t(3t-7)^6$. Using any method you like, find the instantaneous velocity (in metres per second) of the object at time t. At which time(s) is the velocity of the object equal to zero?

$\overline{[50]}$

MATHEMATICS 249 MIDTERM Winter 2003

SHOW ALL WORK. Marks for each problem are to the left of the problem number.
NO CALCULATORS PLEASE.

[4] 1. Find $\lim_{x\to\infty}\left(\dfrac{2-5x^2}{22-x^2+5x}\right)$.

[5] 2. Find $\lim_{x\to 5}\left(\dfrac{5-x}{x^2-2x-15}\right)$.

[5] 3. Find and simplify $\lim_{x\to -3}\left(\dfrac{4-\sqrt{7-3x}}{x^2+3x}\right)$.

[5] 4. Find and simplify $\dfrac{d}{dx}\left(\sqrt{\sin^3 x\ -\ 4}\ \right)$.

[5] 5. Find and simplify $\dfrac{d}{dx}\left(x^{3/5}-\tan(x^5-3)\right)$.

[5] 6. Find and simplify $\dfrac{d}{dx}\left(\dfrac{2-3x}{(x+1)^2}\right)$.

[5] 7. USE THE DEFINITION OF DERIVATIVE to find $\dfrac{d}{dx}(x-x^2)$.

[5] 8. Use implicit differentiation to find and simplify dy/dx where $2xy^2 = x^2 - y^3$.

[6] 9. Find the equation of the tangent line to the graph of $y = 4x^3 + x^{-1}$ at the point where $x = -1$.

[5] 10. An object moves along a straight line so that its position (in metres) at any time $t > 0$ (in seconds) is given by the function $s(t) = kt^3 + t^{-1}$, where k is a constant. The instantaneous velocity of the object at time $t = 1/2$ is 5 metres per second. Find k. Then find the acceleration of the object at time $t = 1/2$.

$\overline{[50]}$