The University of Calgary
Department of Mathematics and Statistics
MATH 249- 03
Quiz #5W

Fall 2005

Name:____________________________**I.D.#:**__________________

1. Solve for x : $\quad \dfrac{1}{2^{2x+1}} = \dfrac{5}{4^{3x}}$. [3]

2. For $f(x) = x^{\sqrt{x}} + \ln(2x)$ find the domain and $f'(x)$. [4]

3. After 2 days a sample of radon decayed to 50 kg.After 4 days to 35 kg.
   What was the original amount? [3]

SOLUTION

For 1)

cross multiply first $\quad 4^{3x} = 5 \cdot 2^{2x+1}$, then apply ln to both sides

thus

$3x \ln 4 = \ln 5 + (2x+1)\ln 2 \qquad 3x \ln 4 - 2x \ln 2 = \ln 5 + \ln 2 = \ln(5 \cdot 2)$

$3x \ln 2^2 - 2x \ln 2 = (6x - 2x)\ln 2 = 4x \ln 2 = \ln 10$ or $x(3\ln 4 - 2\ln 2) = x \ln \frac{4^3}{2^2}$

and finally $x \ln 4^2 = \ln 10 \qquad x = \dfrac{\ln 10}{\ln 16} = \dfrac{\ln 10}{4 \ln 2}$

OR $\quad \dfrac{4^{3x}}{2^{2x+1}} = \dfrac{2^{6x}}{2^{2x+1}} = 2^{4x-1} = 5 \qquad 2^{4x} = 10$

and then log.f. $\quad 4x = \log_2 10 \qquad x = \frac{1}{4}\log_2 10$

For 2)

for $x > 0 \qquad$ rewrite $x^{\sqrt{x}} = e^{\sqrt{x}\ln x}$

$$f'(x) = \left(e^{\sqrt{x}\ln x}\right)' + (\ln 2x)' = e^{\sqrt{x}\ln x}\left(\sqrt{x}\ln x\right)' + \frac{1}{2x} \cdot 2 =$$

$$= e^{\sqrt{x}\ln x}\left(\frac{1}{2\sqrt{x}} \cdot \ln x + \frac{\sqrt{x}}{x}\right) + \frac{1}{x} = e^{\sqrt{x}\ln x}\left(\frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}}\right) + \frac{1}{x}$$

also you may simplify $\ln(2x) = (\ln 2 + \ln x)$

For 2)

the correct formula for the amount of radon $\qquad A(t) = A_0 e^{kt}$ where $k < 0$

t in days ,the initial amount $A_0 = ?$

given $\quad$ if $t = 2 \quad 50 = A_0 e^{2k}$ if $t = 4 \quad 35 = A_0 e^{4k}$

so $\quad A_0 = \dfrac{50}{e^{2k}} = \dfrac{35}{e^{4k}}$ solve for $k \qquad e^{4k-2k} = \frac{35}{50} \qquad e^{2k} = \frac{7}{10}$

$k = \frac{\ln 0.7}{2} = -0.1783374$ and $A_0 = \dfrac{50}{e^{2k}} = 50 \cdot \frac{10}{7} = 71.4286 kg$