MATH 251/249
Worksheet #1 *Solutions.*

For 1 a) $|2x+1| \le |x-2|$
Since $|...|$ is always positive or zero we can square both sides and the sign of the inequality stays the same: $(2x+1)^2 \le (x-2)^2$ since $|...|^2 = (...)^2$ Now
$4x^2+4x+1 \le x^2-4x+4$,everything on one side: $3x^2+8x-3 \le 0$
$(3x-1)(x+3) \le 0$,thus split points are: $x=-3, \frac{1}{3}$,
testing: $--^{pos}---_{3}--^{neg}--_{\frac{1}{3}}--^{pos}---$
so the solutions set :$\left[-3, \frac{1}{3}\right]$
b) $\dfrac{3}{x+1} > \dfrac{1}{3}$ for $x \ne -1$
everything on one side and common denominator: $\dfrac{3\cdot 3-(x+1)}{(x+1)3} > 0$,simplify:
$\dfrac{9-x-1}{3(x+1)} > 0$ then $\dfrac{8-x}{(x+1)(3)} > 0$.So split points are : $x = 8, -1$
testing: $--^{neg}---_{-1}--^{pos}--_{8}--^{neg}---$
solution set: $(-1, 8)$ or $]-1, 8[$.
For 2) $x^2+4x+y^2-2y=11$ $x^2+4x+4+y^2-2y+1 = 11+4+1$
Complete the squares: $(x+2)^2+(y-1)^2 = 16$ so $(x+2)^2+(y-1)^2 = 4^2$
thus $r=4$ and the point $C(-2,1)$ is the centre.

For 3a) $|x+1|+2 > 0$
Since $|...|$ is always positive or zero $|x+1|+2$ is always positive ,so solution set $(-\infty, +\infty)$
b) $\dfrac{3}{x+1} \ge \dfrac{2}{x+3}$ for $x \ne -1, -3$
everything on one side and common denominator: $\dfrac{3(x+3)-2(x+1)}{(x+1)(x+3)} \ge 0$,simplify:
$\dfrac{3x+9-2x-2}{(x+1)(x+3)} \ge 0$ then $\dfrac{(x+7)}{(x+1)(x+3)} \ge 0$.So split points are : $x=-7,-3,-1$
testing: $-^{neg}-_{-7}---^{pos}---_{-3}--^{neg}--_{-1}--^{pos}---$
solution set: $[-7,-3) \cup (-1, +\infty)$ or $[-7,-3[\cup]-1, +\infty[$.
For 4) For $l_1: 3x+2y=1$ $l_2: 2y-3x=0$ $l_3: 3x-2y=0$ and $l_4: 2x-3y=2$
find the slopes: $m_1 = -\frac{3}{2}$,$m_2 = \frac{3}{2}, m_3 = \frac{3}{2}, m_4 = \frac{2}{3}$
so $l_2 \| l_3$ since they have the same slope
and $l_1 \perp l_4$ since $m_1 \cdot m_4 = -1$.

For 5 a) $\dfrac{1}{x+1} \le 1+x$ for $x \ne -1$
everything on one side and common denominator: $\dfrac{1-(x+1)^2}{(x+1)} \le 0$,simplify:
$\dfrac{1-x^2-2x-1}{(x+1)} \le 0$ then $\dfrac{-x(x+2)}{(x+1)} \le 0$.So split points are : $x=0,-2,-1$
testing $-^{pos}--_{-2}--^{neg}-_{-1}--^{pos}---_{0}--^{neg}--$
solution set: $[-2,-1) \cup [0, +\infty)$
b) $|3x-2| > 0$

Since |...| is always positive or zero we have to elliminate zero :$3x-2=0$ for $x=\frac{2}{3}$
The solutions : $x\neq\frac{2}{3}$ or $\left(-\infty,\frac{2}{3}\right)\cup\left(\frac{2}{3},+\infty\right)$
For 6) line perpendicular to the x-axis passing through the point $(-1,3)$.
$\perp$ to x-axis means a vertical line so $x=-1$ (y is any).

For 7 a) $3x+7>x^2$
Everything on one side: $0>x^2-3x-7$ now find the roots ,first
discriminant $D=(-3)^2-4\cdot1\cdot(-7)=9+28=37$, so using the formula roots are
$x_1=\frac{3-\sqrt{37}}{2}=-1.54$ and $x_2=\frac{3+\sqrt{37}}{2}=4.54$
Now testing : $--^{pos}--_{x_1}--^{neg}---_{x_2}--^{pos}-$
OR parabola open up is below the x-axis if $x\in(x_1,x_2)=]-1.54,4.54[$.
b) $\dfrac{x}{2}<\dfrac{2}{x+3}$. for $x\neq-3$
everything on one side and common denominator: $\dfrac{x(x+3)-2\cdot2}{2(x+3)}<0$,simplify:
$\dfrac{x^2+3x-4}{2(x+3)}<0$ then $\dfrac{(x+4)(x-1)}{2(x+3)}<0$.So split points are : $x=-4,-3,1$
testing:$-^{neg}--_{-4}--^{pos}--_{-3}-^{neg}--_{1}-^{pos}--$solution set: $(-\infty,-4)\cup(-3,1)$.
For 8) $x^2-6x+y^2=7$ $x^2+y^2+2y=15$
Complete squares : $(x-3)^2-9+y^2=7, x^2+(y+1)^2-1=15$ SO the equations are:
$(x-3)^2+y^2=16, x^2+(y+1)^2=16$ thus radii are the same $r=4$,
the centres are points $(3,0)$ and $(0,-1)$.
Review Questions.
For 9) for $h\neq0,7$ find common denominator first
$$\frac{\frac{3h+4}{7-h}-\frac{4}{7}}{\frac{25h}{7}}=\frac{\frac{7(3h+4)-4(7-h)}{7(7-h)}}{\frac{25h}{7}}=\frac{21h+28-28+4h}{7(7-h)}\cdot\frac{7}{25h}=\frac{1}{7-h}.$$
For 10) for $x\neq-1$
$$\frac{1}{1+\frac{1}{x+1}}=\frac{1}{\frac{x+1+1}{x+1}}=\frac{x+1}{x+2}\text{ and for }x\neq-2.$$
For 11) factor out the polynomials
$$\frac{x^3+5x^2+6x}{12+x-x^2}=\frac{x(x^2+5x+6)}{-(x^2-x-12)}=\frac{x(x+3)(x+2)}{-(x-4)(x+3)}=\frac{x(x+2)}{-(x-4)}$$
for $x\neq-3,-4$.
For 12) factor out the polynomials ,then common denom.
$$\frac{x}{x^2+x-2}-\frac{2}{x^2-5x+4}=\frac{x}{(x+2)(x-1)}-\frac{2}{(x-4)(x-1)}=\frac{x(x-4)-2(x+2)}{(x+2)(x-1)(x-4)}=$$
$$=\frac{x^2-6x-4}{(x+2)(x-1)(x-4)}\text{ for }x\neq-2,1,4.$$