

The University of Calgary
 Department of Mathematics and Statistics
 MATH 249//03
 Quiz # 4W

FALL 2005

Name: _____ I.D.#: _____

1. Find an equation of the tangent to

$$xy^2 + y = \frac{2x^3}{y} - 1$$

at the point $(-1, 2)$. [4]

2. Find a general antiderivative of $f(x) = \frac{4\sqrt{x} - x^3 - 2}{2x^2}$ for $x > 0$. [3]

3. Solve $y' = 2 \cos(\frac{x}{2} + \pi)$, $y(-\frac{\pi}{2}) = 0$. [3]

Solution For 1)

an equation is $y = m(x + 1) + 2$ for m use the implicit differentiation

$$(xy^2)' + (y)' = 2 \left(\frac{x^3}{y} \right)' - 0 \quad \text{product and quotient rules}$$

$$y^2 + x2yy' + y' = 2 \cdot \frac{3x^2y - x^3y'}{y^2} \quad \text{now } x = -1, y = 2, y' = m$$

$$4 - 4m + m = 2 \cdot \frac{6 + m}{4} \quad 4 - 3m = 3 + \frac{1}{2}m \quad 1 = \frac{7}{2}m$$

so $m = \frac{2}{7}$ and an equation is $y = \frac{2}{7}(x + 1) + 2$ or $7y - 2x = 16$

For 2)

$$\int \frac{4\sqrt{x} - x^3 - 2}{2x^2} dx = 2 \int \frac{\sqrt{x}}{x^2} dx - \frac{1}{2} \int \frac{x^3}{x^2} dx - \int \frac{1}{x^2} dx =$$

$$= 2 \int x^{-\frac{3}{2}} dx - \frac{1}{2} \int x dx - \int x^{-2} dx = -4x^{-\frac{1}{2}} - \frac{1}{4}x^2 + x^{-1} + c, x > 0.$$

For 3)

$$y = \int y' dx = 2 \int \cos(\frac{x}{2} + \pi) dx = 4 \sin\left(\frac{x}{2} + \pi\right) + c$$

using $\int \cos(ax + b) dx = \frac{1}{a} \sin(ax + b) + c$ where $a = \frac{1}{2}$

now, $y = 0, x = -\frac{\pi}{2}, c = ?$

$$0 = 4 \sin\left(-\frac{\pi}{4} + \pi\right) + c = 4 \sin\left(\frac{3\pi}{4}\right) + c = 2\sqrt{2} + c \text{ and } c = -2\sqrt{2} \text{ finally,}$$

$$y = 4 \sin\left(\frac{x}{2} + \pi\right) - 2\sqrt{2}$$

$$\text{ALSO, } \cos\left(\frac{x}{2} + \pi\right) = -\cos \frac{x}{2} \text{ so } y = -2 \int \cos\left(\frac{x}{2}\right) dx = -4 \sin \frac{x}{2} + c$$