The University of Calgary
Department of Mathematics and Statistics
MATH 249 Lecture 3
Quiz # 1W

Fall 2005

Name:____________________________**I.D.#:**_________________

1. Solve for x:
   $|x-1| < 2x.$ [4]

   Solution

   Since the left-hand side is always positive or 0 (for $x = 1$) and the right-hand side must must be bigger, $x > 0$

   Then both sides are positive and we can square

   $(x-1)^2 < (2x)^2$ so $x^2 - 2x + 1 < 4x^2$ then $0 < 3x^2 + 2x - 1$

   $(3x-1)(x+1) < 0$ roots=split points $x = \frac{1}{3}, -1$

   testing $--^{pos}--_{-1}--^{neg}--\frac{1}{3}--^{pos}--$

   thus $x > \frac{1}{3}$ or $x < -1$.Together with the original condition $x > 0$ we have $x > \frac{1}{3}$

   $x \in \left]\frac{1}{3}, +\infty\right[$

2. Solve for x: $\dfrac{10}{2-x} \leq x + 3.$ [3]

   Everything on one side and common denominator:$\dfrac{10-(x+3)(2-x)}{(2-x)} \leq 0,$

   for $x \neq 2$ simplify

   $\dfrac{10-(2x+6-3x-x^2)}{(2-x)} \leq 0$ and finally $\dfrac{x^2+x+4}{(2-x)} \leq 0;$

   the discriminant of the quadratic polynomial on the top is $D = (1)^2 - 4 \cdot 4 = -15,$

   thus no real roots and the top is always positive.

   The fraction will be negative (never 0) if the bottom is negative i.e.$2 - x < 0, 2 < x$

   OR the only split point is $x = 2$

   testing: $-^{pos}---\circ_2--^{neg}---$ so $x \in]2, +\infty[$

3. Find an equation of the circle with the center at the y-intercept of the line $2x - 3y = 6$ and radius 2. [3]

We can find the y-intercept by substituting $x = 0$ into the equation of the given line so : $-3y = 6, y = -2$, thus the center is at $C(0, -2)$ and an equation is
$x^2 + (y + 2)^2 = 4$