The University of Calgary
Department of Mathematics and Statistics
MATH 249-03
Quiz # 2R

Fall,2005

Name:______________________________**I.D.#:**__________________

1. For $f(x) = \sqrt{x-3}$ and $g(x) = \dfrac{3}{x+1}$ find $f \circ g$ and its domain. [3]

2. For $\quad f(x) = \dfrac{|x-1| - x + 1}{2x^2 - x - 1} \quad$ find $\lim f(x)$ as

   (a) $x \to 1^-$

   (b) $x \to +\infty$ [4]

3. For $\quad g(x) = \dfrac{1}{x}\left(\dfrac{4}{x+4} - 1\right) \quad$ find $\lim g(x)$ as

   (a) $x \to 0$

   (b) $x \to -4^+$ [3]

SOLUTION

For 1)

domains $\quad D_f = \{x \geq 3\} = [3, +\infty[\quad D_g = \{x \neq -1\}$

$f \circ g(x) = f(g(x)) = \sqrt{(..) - 3} = \sqrt{\dfrac{3}{x+1} - 3} =$

we can simplify $\quad \sqrt{\dfrac{3 - 3(x+1)}{(x+1)}} = \sqrt{\dfrac{3 - 3x - 3}{(x+1)}} = \sqrt{\dfrac{-3x}{(x+1)}}$

for the domain solve $\quad \dfrac{-3x}{x+1} \geq 0$

split points $x = -1, 0$ are switch points

testing$- -^{neg} - -_{-1} -^{pos} - -_{0} -_{neg} --$

therefore $\quad D_{f \circ g} =]-1, 0]$

For 2a)

since $x < 1 \quad x - 1 < 0 \quad |x-1| = -(x-1) = 1 - x$

$.f(x) = \dfrac{|x-1| - x + 1}{2x^2 - x - 1} = \dfrac{1 - x - x + 1}{(2x+1)(x-1)} = \dfrac{2(1-x)}{(2x+1)(x-1)} = \dfrac{-2(x-1)}{(2x+1)(x-1)} =$
$\dfrac{-2}{(2x+1)}$

so

$\lim_{x \to 1^-} f(x) = \frac{-2}{3}$

as $x \to +\infty$ $\quad$ $x >> 1$ and $x - 1 > 0$

$f(x) = \frac{|x-1| - x + 1}{2x^2 - x - 1} = f(x) = \frac{x - 1 - x + 1}{2x^2 - x - 1} = 0$ so $\lim_{x \to +\infty} f(x) = 0$

also from the original

$$\lim_{x \to +\infty} .f(x) = \lim_{x \to +\infty} \frac{|x-1| - x + 1}{2x^2 - x - 1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x \to +\infty} \frac{\left|\frac{1}{x} - \frac{1}{x^2}\right| - \frac{1}{x} + \frac{1}{x^2}}{2 - \frac{1}{x} - \frac{1}{x^2}} = \frac{0}{2} = 0$$

For 3)

simplify first for $x \neq 0, -4$

$$g(x) = \frac{1}{x}\left(\frac{4}{x+4} - 1\right) = g(x) = \frac{1}{x}\left(\frac{4 - (x+4)}{x+4}\right) = \frac{1}{x}\left(\frac{-x}{x+4}\right) = \frac{-1}{x+4}$$

in **a**) **limit** is $-\frac{1}{4}$.

in **b**) $x > -4$ $\quad$ $x + 4 > 0$ $\quad$ so "$\frac{-1}{0^+}$so **limit** is $-\infty$.