The University of Calgary
Department of Mathematics and Statistics
MATH 249-03
Quiz # 2W

Fall 2005

Name:______________________________**I.D.#:**__________________

1. For $f(x) = \dfrac{5}{x-3}$ and $g(x) = \dfrac{x}{x+1}$ find the compositions
   $g \circ f$ and its domain. [3]

2. For $f(x) = \dfrac{3-x}{5x-x^2-6}$ find $\lim f(x)$
   (a) as $x \to 3$; (b) as $x \to 2^-$. [3]

3. For $g(x) = \dfrac{2-\sqrt{x+5}}{x+1}$ find $\lim g(x)$
   (a) as $x \to -1$; (b) as $x \to +\infty$. [4]

SOLUTION

For 1)

First domain of the given functions

for $f(x) = \dfrac{5}{x-3}$ it must $x - 3 \neq 0$ $D_f = \{x \neq 3\}$ and

for $g(x) = \dfrac{x}{x+1}$ it must $x + 1 \neq 0$ $D_g = \{x \neq -1\}$

for $x \neq 3$

$$(g \circ f)(x) = l(...) = \frac{()}{()+1} = \frac{\left(\frac{5}{x-3}\right)}{\left(\frac{5}{x-3}\right)+1} = \frac{\frac{5}{x-3}}{\frac{5+x-3}{x-3}} = \frac{\left(\frac{5}{x-3}\right)(x-3)}{\left(\frac{2+x}{x-3}\right)(x-3)} = \frac{5}{2+x}$$

$D_{g \circ f} = \{x \neq 3, -2\}$

For 2) simplify first for $x \neq 2, 3$

$$f(x) = \frac{3-x}{5x-x^2-6} = f(x) = \frac{3-x}{-(x^2-5x+6)} = f(x) = \frac{x-3}{(x-3)\,(x-2)} = \frac{1}{x-2}$$

for a)

$\lim_{x \to 3} f(x) = \frac{1}{3-2} = 1$

for b) $x < 2$

$$\lim_{x \to 2^-} f(x) = "\frac{1}{0^-}" = -\infty$$

For 3)

g is defined for $x \in [-5, +\infty)$ and $x \neq -1$

since $\sqrt{x+5}$ is defined only if $\qquad x+5 \geq 0$

$$g(x) = \frac{2-\sqrt{x+5}}{x+1} \cdot \frac{2+\sqrt{x+5}}{2+\sqrt{x+5}} = \frac{4-(x+5)}{(x+1)\left(2+\sqrt{x+5}\right)} = \frac{-(x+1)}{(x+1)\left(2+\sqrt{x+5}\right)}$$

and $\lim\limits_{x\to-1} g(x) = \lim\limits_{x\to 1} \dfrac{-1}{\left(2+\sqrt{x+5}\right)} = \dfrac{-1}{4}$

for b) we can use the original or simplified form

$$\lim_{x\to+\infty} g(x) = \lim_{x\to\infty} \frac{-1}{\left(2+\sqrt{x+5}\right)} = \text{"}\tfrac{1}{\infty}\text{"} = 0 \qquad \text{OR}$$

$$\lim_{x\to+\infty} g(x) = \lim_{x\to+\infty} \frac{2-\sqrt{x+5}}{x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to-\infty} \frac{\frac{2}{x} - \frac{\sqrt{x+5}}{x}}{1+\frac{1}{x}} = \lim_{x\to-\infty} \frac{\frac{2}{x} - \frac{\sqrt{x+5}}{\sqrt{x^2}}}{1+\frac{1}{x}} =$$

$$= \lim_{x\to-\infty} \frac{\frac{2}{x} - \sqrt{\frac{x+5}{x^2}}}{1-\frac{1}{x}} = \lim_{x\to-\infty} \frac{\frac{2}{x} + \sqrt{\frac{1}{x} + \frac{1}{x^2}}}{1-\frac{1}{x}} = \tfrac{0}{1} = 0.$$