The University of Calgary
Department of Mathematics and Statistics
MATH 249-03
Quiz # 4R

FALL 2005

Name:______________________________**I.D.#:**__________________

1. Find an equation of the tangent to
$$\sqrt{2x+y} = \frac{3x}{y} + 9$$
at the point $(6, -3)$. [4]

2. Find a general antiderivative of $f(x) = \sin(2x-3) - \cos(\dfrac{1-x}{3})$ [3]

3. Solve $y'' = \dfrac{2}{x^3} - 6\sqrt{x}$,$y'(1) = 0$, $y(1) = 6$. [3]

Solution

For 1)

an equation $\quad y = m(x-6) - 3$ and to find m use

implicit differentiation $\quad \left[(2x+y)^{\frac{1}{2}}\right]' = 3\left(\dfrac{x}{y}\right)' + 0$

$\dfrac{1}{2}(2x+y)^{-\frac{1}{2}} \cdot (2x+y)' = 3 \cdot \dfrac{y - xy'}{y^2} \qquad (2x+y)^{-\frac{1}{2}}(2+y') = 6 \cdot \dfrac{y-xy'}{y^2}$

now,$x = 6, y = -3, y\prime = m$

$\dfrac{1}{\sqrt{9}}(2+m) = 6\dfrac{-3-6m}{3^2} = \dfrac{-6-12m}{3}$ so $2 + m = -6 - 12m$

and $13m = -8 \qquad y = -\frac{8}{13}(x-6) - 3$ OR $13y + 8x = 9$

For 2)

$\displaystyle\int \sin(2x-3) - \cos(\frac{1-x}{3})dx = \int \sin(2x-3)dx - \int \cos(\frac{1}{3} - \frac{1}{3}x)dx =$

$= \dfrac{-\cos(2x-3)}{2} - \dfrac{\sin\left(\frac{1}{3} - \frac{1}{3}x\right)}{\frac{-1}{3}} + c = \dfrac{-1}{2}\cos(2x-3) + 3\sin\left(\dfrac{1}{3} - \dfrac{1}{3}x\right) + c$

using $\displaystyle\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b) + c$ and $\displaystyle\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b) + c$

For 3)

for $x > 0 \qquad y' = \displaystyle\int y''dx = \int \left(\frac{2}{x^3} - 6\sqrt{x}\right)dx = 2\int x^{-3}dx - 6\int x^{\frac{1}{2}}dx =$

$= 2 \cdot \dfrac{x^{-2}}{-2} - 6 \cdot \dfrac{2}{3}x^{\frac{3}{2}} + c_1 = -x^{-2} - 4x^{\frac{3}{2}} + c_1$

now, $x = 1, y' = 0, c_1 = ?$

$0 = -1 - 4 + c_1$ so $c_1 = 5$ and $y' = -x^{-2} - 4x^{\frac{3}{2}} + 5$

then $\quad y = \int y' dx = \int \left(-x^{-2} - 4x^{\frac{3}{2}} + 5\right) dx = x^{-1} - 4 \cdot \frac{2}{5} x^{\frac{5}{2}} + 5x + c_2$

$y = x^{-1} - \frac{8}{5} x^{\frac{5}{2}} + 5x + c_2$ to find c_2 substitute $x = 1, y = 0$

$6 = 1 - \frac{8}{5} + 5 + c_2 \qquad c_2 = \frac{8}{5}$ and $y = x^{-1} - \frac{8}{5} x^{\frac{5}{2}} + 5x + \frac{8}{5}, x > 0.$