The University of Calgary
Department of Mathematics and Statistics
MATH 249- 03 Quiz #3R

FALL 2005

Name:____________________________**I.D.#:**________________]

1. Using the **definition of the derivative** find $f'(3)$ if $f(x)\sqrt{5-\frac{x}{3}}$. [4]

2. Find $f'(x)$ if $f(x) = \left(\frac{x^4}{4} + \frac{1}{5x}\right) \cdot (5-2x)^3$. [3]

3. Find all points on the graph $y = \sqrt{4x^3 - 6x^2 + 1}$ with horizontal tangents. [3]

For 1) $f(3) = \sqrt{5-1} = 2$

$$f'(3) = \lim_{x\to 3} \frac{f(x)-f(3)}{x-3} = \lim_{x\to 3} \frac{\sqrt{5-\frac{x}{3}}-2}{x-3} \cdot \frac{\sqrt{5-\frac{x}{3}}+2}{\sqrt{5-\frac{x}{3}}+2} = \lim_{x\to 3} \frac{5-\frac{x}{3}-2^2}{x-3} \cdot \frac{1}{\sqrt{5-\frac{x}{3}}+2} =$$

$$= \lim_{x\to 3} \frac{\frac{1}{3}(3-x)}{(x-3)\left(\sqrt{5-\frac{x}{3}}+2\right)} = \lim_{x\to 3} \frac{-\frac{1}{3}}{\sqrt{5-\frac{x}{3}}+2} = -\frac{1}{12} \qquad \text{OR}$$

$$f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{\sqrt{5-\frac{3+h}{3}}-2}{h} = \lim_{h\to 0} \frac{\sqrt{\frac{12-h}{3}}-2}{h} \cdot \frac{\sqrt{\frac{12-h}{3}}+2}{\sqrt{\frac{12-h}{3}}+2} =$$

$$= \lim_{h\to 0} \frac{\frac{12-h}{3}-4}{h\left(\sqrt{\frac{12-h}{3}}+2\right)} = \lim_{h\to 0} \frac{-\frac{h}{3}}{h\left(\sqrt{\frac{12-h}{3}}+2\right)} = \frac{-\frac{1}{3}}{2+2} = -\frac{1}{12}.$$

Check by Rules: $f'(x) = \frac{1}{2}\left(5-\frac{x}{3}\right)^{-\frac{1}{2}} \cdot \left(-\frac{1}{3}\right)$ and at $x=3$,

$f'(2) = \frac{-1}{6} \cdot (5-1)^{-\frac{1}{2}} = -\frac{1}{6} \cdot \frac{1}{\sqrt{4}} = -\frac{1}{12}$.

For 2) For $x \neq 0$ by Product Rule

$$f'(x) = \left(\frac{1}{4}x^4 + \frac{1}{5}x^{-1}\right)' \cdot (5-2x)^3 + \left(\frac{x^4}{4} + \frac{1}{5x}\right) \cdot \left[(5-2x)^3\right]' =$$

and by Chain Rule

$$= \left(\frac{1}{4} \cdot 4x^3 + \frac{1}{5}(-1)x^{-2}\right) \cdot (5-2x)^3 + \left(\frac{x^4}{4} + \frac{1}{5x}\right) \cdot 3(5-2x)^2(-2) =$$

$$= \left(x^3 - \frac{1}{5x^2}\right)(5-2x)^3 - 6(5-2x)^2\left(\frac{x^4}{4} + \frac{1}{5x}\right) = ...$$

For 3)

horizontal tangent means $m = y' = 0$, by Chain Rule

$$y' = (\sqrt{4x^3-6x^2+1})' = \frac{1}{2}(4x^3-6x^2+1)^{-\frac{1}{2}} \cdot (4x^3-6x^2+1)' = \frac{12x^2-12x}{2\sqrt{4x^3-6x^2+1}}$$

$y' = 0$ if the numarator is 0: $12x(x-1) = 0, x = 0, 1$

For $x = 0, y = \sqrt{0+1} = 1$,so the point is $P(0,1)$

for $x = 1, y = \sqrt{4-6+1} = \sqrt{neg}$,so no more points, f is NOT defined at $x = 1$.