The University of Calgary
Department of Mathematics and Statistics
MATH 249- 03 Quiz #3W

FALL 2005

Name:______________________________**I.D.#:**__________________

1. Using the **definition of derivative** find $f'(6)$ if $f(x) = \dfrac{x}{3} - \dfrac{6}{x}$. [3]

2. Find $f'(-1)$ if $f(x) = \dfrac{7x}{\sqrt{2x+6}}$. [4]

3. Find an equation of the tangent line to $y = \sqrt[3]{2x^3 + 3x - 3}$ at $x = -1$. [3]

Solution
For 1)

$f(6) = \frac{6}{3} - 1 = 1 \qquad f'(6) = \lim\limits_{x \to 6} \dfrac{f(x) - f(6)}{x-6} = \lim\limits_{x \to 6} \dfrac{\frac{x}{3} - \frac{6}{x} - 1}{x-6}$

$= \lim\limits_{x \to 6} \dfrac{\frac{x^2 - 18 - 3x}{3x}}{x-6} = \lim\limits_{x \to 6} \dfrac{(x-6)(x+3)}{3x\,(x-6)} = \lim\limits_{x \to 6} \dfrac{(x+3)}{3x} = \dfrac{1}{2}$

OR

$f'(6) = \lim\limits_{h \to 0} \dfrac{f(6+h) - f(6)}{h} = \lim\limits_{h \to 0} \dfrac{\frac{6+h}{3} - \frac{6}{6+h} - 1}{h} = \lim\limits_{h \to 0} \dfrac{1}{h} \left[\dfrac{(6+h)^2 - 18 - 3\,(6+h)}{3\,(6+h)} \right] =$

$= \lim\limits_{h \to 0} \dfrac{1}{h} \left[\dfrac{36 + 12h + h^2 - 18 - 18 - 3h}{3\,(6+h)} \right] = \lim\limits_{h \to 0} \dfrac{1}{h} \left[\dfrac{h(12 + h - 3)}{3\,(6+h)} \right] =$

$= \lim\limits_{h \to 0} \dfrac{(9+h)}{3\,(6+h)} = \frac{9}{18} = \dfrac{1}{2}$

check by Rules $\quad f'(x) = (\dfrac{1}{3}x - 6x^{-1})' = \frac{1}{3} + \frac{6}{x^2}$ and at $x = 6$ we get $\frac{1}{2}$

For 2)

by Quotient and Chain Rules $f'(x) = \left(\dfrac{7x}{\sqrt{2x+6}} \right)' = \dfrac{(7x)'\sqrt{2x+6} - 7x\left[(2x+6)^{\frac{1}{2}}\right]'}{\left(\sqrt{2x+6}\right)^2} =$

$= \dfrac{7\sqrt{2x+6} - 7x\left[\frac{1}{2}\,(2x+6)^{-\frac{1}{2}} \cdot 2\right]}{2x+6} = \dfrac{7\sqrt{2x+6} - 7x\,(2x+6)^{-\frac{1}{2}}}{2x+6}$

at $x = -1$

$f'(-1) = \dfrac{7\sqrt{4} + \frac{7}{\sqrt{4}}}{4} = \frac{7}{4}\left(2 + \frac{1}{2}\right) = \dfrac{35}{8}$

OR by Product and Chain Rules $\quad f"(x) = \left[7x(2x+6)^{-\frac{1}{2}}\right]' = (7x)'\,(2x+6)^{-\frac{1}{2}} + 7x\left[(2x+6)^{-\frac{1}{2}}\right]' =$

$= 7(2x+6)^{-\frac{1}{2}} + 7x\left(-\frac{1}{2}\right)(2x+6)^{-\frac{3}{2}} \cdot 2 = \ldots$

For 3)

for $x = -1 \qquad y = (-8)^{\frac{1}{3}} = -2$ so the point is $P\,(-1, -2)$

an equation $\qquad y = m_t\,(x+1) - 2$

for the slope : by Chain Rule

$$y' = \left[(2x^3+3x-3)^{\frac{1}{3}}\right]' = \tfrac{1}{3}\,(2x^3+3x-3)^{-\frac{2}{3}}\,(6x^2+3) = \frac{2x^2+1}{(2x^3+3x-3)^{\frac{2}{3}}}$$

at $x=-1$ $\quad m_t = \dfrac{3}{[(-8)^{\frac{1}{3}}]^2} = \dfrac{3}{4}$ and $y = \frac{3}{4}\,(x+1) - 2$ or $3x - 4y = 5$