THE UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS
FINAL Handout
MATH 251/249

1. Evaluate the limits:

   (a) (a) $\lim_{x\to\pi} \dfrac{\cos\left(\frac{x}{2}\right)}{\pi - x}$ $\quad(b)\quad \lim_{x\to-\infty} \dfrac{\cos\left(\frac{x}{2}\right)}{\pi - x}$ $\quad(c)\quad \lim_{x\to+\infty} \dfrac{x}{2^x - 1}$

   (d) $\lim_{x\to-\infty} \dfrac{x}{2^x - 1}$ $\quad(e)\quad \lim_{x\to 0} \dfrac{x}{2^x - 1}$

2. Find the domain and the derivative of f of

   (a) $f(x) = \dfrac{x}{3} e^{-\sin\left(\frac{3}{x}\right)}$

   (b) $f(x) = \dfrac{\ln(2x - 3)}{e^{-x^2}}$

3. **A**

   Sketch the graph of $y = \dfrac{1}{x^2 + x - 2}$ if $y' = -\dfrac{2x+1}{(x^2+x-2)^2}$ and $y'' = \dfrac{6x^2+6x+6}{(x^2+x-2)^3}$ i.e.

   (a) find the domain, range, vertical and horizontal asymptotes, x and y intercepts;
   (b) find the intervals where f is increasing or decreasing;
   (c) find the intervals where f is concave down or up

   3B

   Sketch the graph of $y = x(4 - x)^3$.Indicate where the function is increasing,decreasing, concave up,concave down;find the domain and range.

4. (a) Find the tangent approximation (linearization) of

   $$f(x) = \frac{1}{\sqrt{2x^2 + 1}} \text{ around } x_0 = 2.$$

   (b) Use it to estimate $\frac{1}{\sqrt{3}}$.

5. **A**

   Sketch a graph of <u>one</u> function f satisfying all the following conditions:

   (a) f is defined for all x, continuous there except
   (b) f is discontinuous at $x = 2, 4$ where $\lim_{x\to 4^-} f(x) = f(4) = 0$, $\quad x = 2$ is a vertical asymptote.

(c) $y = 3$ is a horizontal asymptote and $\lim_{x\to-\infty} f(x)$ does not exist,

(d) f is increasing on $]3,4[$ and on $]4,+\infty[$, f is decreasing on $]0,2[$ and on $]2,3[$, and $f'(x) = 0$ for all $x \in]-2,0[$;

(e) f is concave up on $]0,1[$ and on $]3,4[$; f is concave down on $]1,2[$, on $]2,3[$ and on $]4,+\infty[$;

(f) absolute maximum value is 6, and local minimum value is -2.

B

Sketch a graph of one function f satisfying all the following conditions:

(a) f is defined on $]-\infty,1[$, continuous there except at $-2,-4$;

(b) f is discontinuous at $x = -4$ where $\lim_{x\to-4} f(x)$ DNE (does not exist)

(c) $x = -2$ is a vertical asymptote; $y = 3$ is a horizontal asymptote;

(d) f is increasing on $]-3,-2[$and on $]-2,0[$
f is decreasing on $]-\infty,-4[$and on $]-4,-3[$
and $f'(x) = 0$ for all $x \in]0,1[$;

(e) f is concave up on $]-4,-3[$ and on $]-1,0[$;
f is concave down on $]-\infty,-4[$, on$]-3,-2[$ and on $]-2,-1[$;

(f) absolute maximum value is 4, and local minimum value is -3.

6. **A**

A box with a square base(bottom) and NO top(lid) has a volume of 9 m^3.Find the dimensions of the most economical box if the material for the base costs \$2 per m^2 and the material for the sides \$3 per m^2.

B

A landscape architect plans to enclose a 280 m^2 rectangular region in a botanical garden. She will use shrubs costing \$25.00 per meter along three sides and fencing costing \$10.00 per meter along the fourth side. Find the dimensions of the region to minimize the total cost.

Draw a diagram and name the variables

7. Find (a) for $x > 0$ $\int \frac{3\sqrt{x}-5}{x\sqrt{x}}dx =$ (b) $\int 2x^3\sqrt{2x^2+3}dx.$

8. Evaluate (a) $\int_2^3 x2^{x^2}dx$ (b) $\int_0^1 \frac{4x+3}{3-2x}dx.$