✓QUICK CHECK ANSWERS 3.8

1. tangent; $f(x)$; x_0 **2.** $y = 1 + (-4)(x-2)$ or $y = -4x + 9$ **3.** $dy = -0.4$, $\Delta y = -0.41$ **4.** within $\pm 1\%$

CHAPTER REVIEW EXERCISES

Graphing Utility C CAS

1. Explain the difference between average and instantaneous rates of change, and discuss how they are calculated.

2. Complete each part for the function $y = \frac{1}{2}x^2$.
(a) Find the average rate of change of y with respect to x over the interval $[3, 4]$.
(b) Find the instantaneous rate of change of y with respect to x at $x = 3$.
(c) Find the instantaneous rate of change of y with respect to x at a general x-value.
(d) Sketch the graph of $y = \frac{1}{2}x^2$ together with the secant line whose slope is given by the result in part (a), and indicate graphically the slope of the tangent line that corresponds to the result in part (b).

3. Complete each part for the function $f(x) = x^2 + 1$.
(a) Find the slope of the tangent line to the graph of f at a general x-value.
(b) Find the slope of the tangent line to the graph of f at $x = 2$.

4. A car is traveling on a straight road that is 120 mi long. For the first 100 mi the car travels at an average velocity of 50 mi/h. Show that no matter how fast the car travels for the final 20 mi it cannot bring the average velocity up to 60 mi/h for the entire trip.

5. At time $t = 0$ a car moves into the passing lane to pass a slow-moving truck. The average velocity of the car from $t = 1$ to $t = 1 + h$ is

$$v_{\text{ave}} = \frac{3(h+1)^{2.5} + 580h - 3}{10h}$$

Estimate the instantaneous velocity of the car at $t = 1$, where time is in seconds and distance is in feet.

6. A sky diver jumps from an airplane. Suppose that the distance she falls during the first t seconds before her parachute opens is $s(t) = 976((0.835)^t - 1) + 176t$, where s is in feet. Graph s versus t for $0 \le t \le 20$, and use your graph to estimate the instantaneous velocity at $t = 15$.

7. A particle moves on a line way from its initial position so that after t hours it is $s = 3t^2 + t$ miles from its initial position.
(a) Find the average velocity of the particle over the interval $[1, 3]$.
(b) Find the instantaneous velocity at $t = 1$.

8. State the definition of a derivative, and give two interpretations of it.

9. Use the definition of a derivative to find dy/dx, and check your answer by calculating the derivative using appropriate derivative formulas.
(a) $y = \sqrt{9 - 4x}$ (b) $y = \dfrac{x}{x+1}$

10. Suppose that $f(x) = \begin{cases} x^2 - 1, & x \le 1 \\ k(x-1), & x > 1. \end{cases}$
For what values of k is f
(a) continuous? (b) differentiable?

11. The accompanying figure shows the graph of $y = f'(x)$ for an unspecified function f.
(a) For what values of x does the curve $y = f(x)$ have a horizontal tangent line?
(b) Over what intervals does the curve $y = f(x)$ have tangent lines with positive slope?
(c) Over what intervals does the curve $y = f(x)$ have tangent lines with negative slope?
(d) Given that $g(x) = f(x)\sin x$, find $g''(0)$.

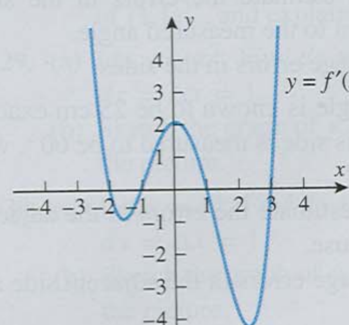

Figure Ex-11

12. Sketch the graph of a function f for which $f(0) = 1$, $f'(0) = 0$, $f'(x) > 0$ if $x < 0$, and $f'(x) < 0$ if $x > 0$.

13. According to the U.S. Bureau of the Census, the estimated and projected midyear world population, N, in billions for the years 1950, 1975, 2000, 2025, and 2050 was 2.555, 4.088, 6.080, 7.841, and 9.104, respectively. Although the increase in population is not a continuous function of the time t, we can apply the ideas in this section if we are willing to approximate the graph of N versus t by a continuous curve, as shown in the accompanying figure.
(a) Use the tangent line at $t = 2000$ shown in the figure to approximate the value of dN/dt there. Interpret your result as a rate of change.
(b) The instantaneous ***growth rate*** is defined as

$$\frac{dN/dt}{N}$$