University of Calgary
Faculty of Science
Department of Mathematics and Statistics

Math 249-L05 Fall 2005

Sample Problem 1.

Determine the centre of the circum-circle of the triangle ABC when A has coordinates (-3,1), B has coordinates (1,5) and C has coordinates (4,-1).

Solution:
The circum-circle of the triangle is the circle which passes through each of the vertices of the triangle.
The centre of the circum-circle is the point of intersection of the perpendicular bisectors of the sides of the triangle.
To solve the problem, we proceed as follows:
Let D and E be the midpoints of the sides AB and BC respectively.

Then, $D = \left(\frac{-3 + 1}{2}, \frac{1 + 5}{2} \right) = (-1, 3)$

and $E = \left(\frac{1 + 4}{2}, \frac{5 + (-1)}{2} \right) = \left(\frac{5}{2}, 2 \right)$

The slope of the straight lines AB and BC are found as follows:

$$m_{BC} = \frac{5 - (-1)}{1 - 4} = \frac{6}{-3} = -2$$

$$m_{AB} = \frac{5 - 1}{1 - (-3)} = 1$$

Let K be the centre of the circum-circle. Then DK is perpendicular to AB and EK is perpendicular to BC.

$$m_{DK} = -1 \text{ and } m_{EK} = \frac{1}{2}$$

We can now determine the equations of the perpendicular lines as follows:
The equation of DK is given by:

$$y - 3 = -1(x + 1)$$
$$\therefore \quad y = -x + 2$$

the equation of EK is given by:

$$y - 2 = \frac{1}{2}\left(x - \frac{5}{2}\right) = \frac{x}{2} - \frac{5}{4}$$

$$\therefore \quad 4y - 8 = 2x - 5$$

$$\therefore \quad 4y = 2x + 3$$

We determine the coordinates of the point of intersection, K, by solving the system of equations:

$$y = -x + 2 \qquad (1)$$

$$4y = 2x + 3 \qquad (2)$$

By using (1) and substituting for y in (2), we get,

$$4(-x + 2) = 2x + 3$$

$$\therefore \quad -4x + 8 = 2x + 3$$

$$\therefore \quad -6x = -5$$

$$\therefore \quad x = \frac{5}{6}$$

From (1), we get,

$$y = -\frac{5}{6} + 2 = \frac{7}{6}$$

Therefore, $K = \left(\frac{5}{6}, \frac{7}{6}\right)$.

Since K is the centre of the circum-circle, the radius of the circle can be found by computing the length of AK, BK or CK.

University of Calgary
Faculty of Science
Department of Mathematics and Statistics

Math 249-L05 Fall 2005

Sample Problem 1.

$$\overline{AK} = \sqrt{\left(\frac{5}{6} - (-3)\right)^2 + \left(\frac{7}{6} - 1\right)^2} = \sqrt{\left(\frac{23}{6}\right)^2 + \frac{1}{36}}$$

$$\therefore \overline{AK} = \sqrt{\frac{530}{36}} = \sqrt{\frac{265}{18}}$$

We can now write the equation of the circle since we have the centre and the radius.

The required circle has equation:

$$\left(x - \frac{5}{6}\right)^2 + \left(y - \frac{7}{6}\right)^2 = \frac{265}{18}$$