University of Calgary
Faculty of Science
Department of Mathematics and Statistics

Math 249

Fall 2005

Worksheet 3 (Answers)

1. Find the domain and range of each of the relations. Say whether or not the relation is a function and give reasons for your answer.

a. $f(x) = \sqrt{x^2 - 4}$
$Dom\ f = (-\infty, -2] \cup [2, \infty)$
$Range\ f = [0, \infty)$
f is a function.

b. $f(x) = \sqrt{6x - x^2}$
$Dom\ f = [0, 6]$
$Range\ f = [0, 3]$
f is a function.

c. $f = \{(x, y) : x^2 - y^2 = 16\}$
$Dom\ f = (-\infty, -4] \cup [4, \infty)$
$Range\ f = [0, \infty)$
f is not a function.

d. $f(x) = \dfrac{|2x - 3|}{6 - 4x}$
$Dom\ f = \left(-\infty, \dfrac{3}{2}\right) \cup \left(\dfrac{3}{2}, \infty\right)$
$Range\ f = \left\{-\dfrac{1}{2}, \dfrac{1}{2}\right\}$
f is a function.

e. $f(x) = \dfrac{1}{x - 4}$
$Dom\ f = (-\infty, 4) \cup (4, \infty)$
$Range\ f = (-\infty, 0) \cup (0, \infty)$
f is a function.

University of Calgary
Faculty of Science
Department of Mathematics and Statistics

Math 249

Fall 2005

Worksheet 3 (Answers)

f. $f(x) = \dfrac{1}{(x - 2)^2}$

Dom $f = (-\infty, 2) \cup (2, \infty)$

Range $f = (0, \infty)$

f is a function.

g. $f(x) = \sqrt{2 - \sqrt{x}}$

h. $f = \{(x, y) : x^2 + y^2 = 4\}$

This is a circle which has centre at (0,0).

Dom $f = [-2, 2]$

Range $f = [-2, 2]$

f is not a function.

2. Find each of the following limits if they exist. If they do not exist, give reasons for your answers.

a. $\lim\limits_{x \to -2} (3x^2 - 2x + 7) = 23$

b. $\lim\limits_{x \to 2} \left(4x^2 - \dfrac{2}{x}\right) = 15$

c. $\lim\limits_{x \to 2} \left(\dfrac{x^2 + x - 6}{x^2 + 3x - 10}\right) = \dfrac{5}{7}$

d. $\lim\limits_{x \to 1} \left(\dfrac{\sqrt{3x + 4} - \sqrt{5x + 2}}{\sqrt{2x^2 + 7x} - 3}\right) = -\dfrac{6}{11\sqrt{7}}$

e. $\lim\limits_{x \to 2} \left(\dfrac{4x - 8}{\sqrt{2x + 5} - \sqrt{x^2 + 5}}\right) = -12$

University of Calgary
Faculty of Science
Department of Mathematics and Statistics

Math 249

Fall 2005

Worksheet 3 (Answers)

f. $\lim_{x \to 3} \left(\frac{|x - 3|}{x - 3} \right)$ does not exist since the left and write limits are not equal.

g. $\lim_{x \to 0} \left(\frac{1}{x\sqrt{1 + x}} - \frac{1}{x} \right) = -\frac{1}{2}$

h. $\lim_{x \to 0^-} \left(\frac{1}{x} - \frac{1}{|x|} \right) = -\infty$

i. $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{|x|} \right) = 0$

j. $\lim_{x \to 1} \left(\frac{x^2 - 1}{|x - 1|} \right)$ does not exist since the left and write limits are not equal.

k. $\lim_{x \to 2} \left(\frac{\sqrt{6 - x} - 2}{\sqrt{3 - x} - 1} \right) = \frac{1}{2}$

3. Find a so that $\lim_{x \to -2} f(x)$ exists when $f(x) = \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$

$a = 15$.

$\lim_{x \to -2} f(x) = -1$.