University of Calgary
Faculty of Science
Department of Mathematics and Statistics

Math 249 Fall 2005

Worksheet 5 [Answers]

1. Find each of the following limits if they exist. If the limit does not exist, explain why.

a. $\lim_{x\to\infty}\left(\frac{-3x^2+x-6}{x^2+3x-10}\right) = -3$

b. $\lim_{x\to-\infty}\left(\frac{7-3x^2-6x^3}{4x^2+3x-10}\right) = -\infty$

c. $\lim_{x\to\infty}\left(\frac{1}{x}+1\right)\left(\frac{5x^2-7}{x^2}\right) = 5$

d. $\lim_{x\to\infty}\left(\frac{x^2+x-6}{4x^3+3x-10}\right) = 0$

e. $\lim_{x\to-\infty}\left(\frac{3x^2+x-7}{10-4x-5x^2}\right) = -\frac{3}{5}$

f. $\lim_{x\to-\infty}\left(\frac{3x^3+5x^2-7}{10x^3-11x+5}\right) = \frac{3}{10}$

g. $\lim_{x\to-\infty}|x| = \infty$

h. $\lim_{x\to\infty}\frac{|x|}{|x|+1} = 1$

i. $\lim_{x\to\infty}\left(\sqrt{x^2+1}-x\right) = 0$

j. $\lim_{x\to\infty}\left(\sqrt{3x^2+8x+6}-\sqrt{3x^2+3x+1}\right) = \frac{5}{2\sqrt{3}}$

k. $\lim_{x\to\infty}\left(\sqrt{\frac{12x^3-5x+2}{1+4x^2+3x^3}}\right) = 2$

2. Find each limit if they exist. If the limit does not exist, explain why.

a. $\lim_{x\to0}\left(\frac{\sin(3x)}{2x}\right) = \frac{3}{2}$

b. $\lim_{x\to0}\left(\frac{\tan 4x}{\sin 3x}\right) = \frac{4}{3}$

c. $\lim_{x\to0}\left(\frac{1-\cos(2x)}{4x^2}\right) = \frac{1}{2}$

d. $\lim_{x\to\infty}\left(\frac{x+\sin x}{x+\cos x}\right) = 1$
[Use the squeeze theorem]

e. $\lim_{x\to\infty}\left(1+\cos\left(\frac{1}{x}\right)\right) = 2$

f. $\lim_{x\to0}\left(\frac{\sin x-\sin x\cos x}{x^2}\right) = 0$

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g. $\lim_{x \to 0} (x \cot x) = 1$ h. $\lim_{x \to 0} \left(\frac{\sin(\alpha + x) - \sin \alpha}{x} \right) = \cos \alpha$

i. $\lim_{x \to 0} \left(\frac{\cos(\alpha + x) - \cos \alpha}{x} \right) = -\sin \alpha$

j. $\lim_{x \to 0} \left(\frac{\tan(\alpha + x) - \tan \alpha}{x} \right) = \sec^2 \alpha$

3. Use the Squeeze Theorem to evaluate the following limits :

a. $\lim_{x \to \infty} \left(\frac{\sin(x)}{x} \right) = 0$ b. $\lim_{x \to 0} \left(\sqrt{x^3 + x^2} \sin\left(\frac{\pi}{x} \right) \right) = 0$

c. $\lim_{x \to 0} \left(x^4 \cos\left(\frac{2}{x} \right) \right) = 0$

4. Determine the horizontal asymptotes of the graph of each of the functions given below:

a. $y = \frac{\sqrt{3x^2 + 1}}{4x - 5}$ Horizontal asymptotes are $y = \frac{\sqrt{3}}{4}$ and $y = -\frac{\sqrt{3}}{4}$

b. $y = \frac{x - 9}{\sqrt{4x^2 + 3x + 2}}$ Horizontal asymptotes are $y = \frac{1}{2}$ and $y = -\frac{1}{2}$

5. Show that the cubic equation $x^3 + x^2 - x - 4 = 0$ has a root in the interval (1,2).

6. If $f(x) = x^3 + x - 1$, show that f has a zero between $x = 0$ and $x = 1$.

7. Show that $f(x) = x^3 - 15x + 1$ has at least three zeros in the closed interval $[-4, 4]$.