

Worksheet 5

1. Find each of the following limits if they exist. If the limit does not exist, explain why.

a.
$$\lim_{x \rightarrow \infty} \left(\frac{-3x^2 + x - 6}{x^2 + 3x - 10} \right)$$

b.
$$\lim_{x \rightarrow \infty} \left(\frac{7 - 3x^2 - 6x^3}{4x^2 + 3x - 10} \right)$$

c.
$$\lim_{x \rightarrow \infty} \left(\frac{1}{x} + 1 \right) \left(\frac{5x^2 - 7}{x^2} \right)$$

d.
$$\lim_{x \rightarrow \infty} \left(\frac{x^2 + x - 6}{4x^3 + 3x - 10} \right)$$

e.
$$\lim_{x \rightarrow -\infty} \left(\frac{3x^2 + x - 7}{10 - 4x - 5x^2} \right)$$

f.
$$\lim_{x \rightarrow -\infty} \left(\frac{3x^3 + 5x^2 - 7}{10x^3 - 11x + 5} \right)$$

g.
$$\lim_{x \rightarrow -\infty} |x|$$

h.
$$\lim_{x \rightarrow \infty} \frac{|x|}{|x| + 1}$$

i.
$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + 1} - x)$$

j.
$$\lim_{x \rightarrow \infty} (\sqrt{3x^2 + 8x + 6} - \sqrt{3x^2 + 3x + 1})$$

k.
$$\lim_{x \rightarrow \infty} \left(\sqrt{\frac{12x^3 - 5x + 2}{1 + 4x^2 + 3x^3}} \right)$$

2. Find each limit if they exist. If the limit does not exist, explain why.

a.
$$\lim_{x \rightarrow 0} \left(\frac{\sin(3x)}{2x} \right)$$

b.
$$\lim_{x \rightarrow 0} \left(\frac{\tan 4x}{\sin 3x} \right)$$

c.
$$\lim_{x \rightarrow 0} \left(\frac{1 - \cos(2x)}{4x^2} \right)$$

d.
$$\lim_{x \rightarrow \infty} \left(\frac{x + \sin x}{x + \cos x} \right)$$

e.
$$\lim_{x \rightarrow \infty} \left(1 + \cos \left(\frac{1}{x} \right) \right)$$

f.
$$\lim_{x \rightarrow 0} \left(\frac{\sin x - \sin x \cos x}{x^2} \right)$$

g.
$$\lim_{x \rightarrow 0} (x \cot x)$$

h.
$$\lim_{x \rightarrow 0} \left(\frac{\sin(\alpha + x) - \sin \alpha}{x} \right)$$

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i. $\lim_{x \rightarrow 0} \left(\frac{\cos(\alpha + x) - \cos \alpha}{x} \right)$ j. $\lim_{x \rightarrow 0} \left(\frac{\tan(\alpha + x) - \tan \alpha}{x} \right)$

3. Use the Squeeze Theorem to evaluate the following limits :

a. $\lim_{x \rightarrow \infty} \left(\frac{\sin(x)}{x} \right)$ b. $\lim_{x \rightarrow 0} \left(\sqrt{x^3 + x^2} \sin \left(\frac{\pi}{x} \right) \right)$

c. $\lim_{x \rightarrow 0} \left(x^4 \cos \left(\frac{2}{x} \right) \right)$

4. Determine the horizontal asymptotes of the graph of each of the functions given below:

a. $y = \frac{\sqrt{3x^2 + 1}}{4x - 5}$

b. $y = \frac{x - 9}{\sqrt{4x^2 + 3x + 2}}$

5. Show that the cubic equation $x^3 + x^2 - x - 4 = 0$ has a root in the interval (1,2).

6. If $f(x) = x^3 + x - 1$, show that f has a zero between $x = 0$ and $x = 1$.

7. Show that $f(x) = x^3 - 15x + 1$ has at least three zeros in the closed interval $[-4, 4]$.