University of Calgary
Faculty of Science
Department of Mathematics and Statistics

11.1

Math 249-L05

Fall 2005

Worksheet 12 Solutions
[More Integration with applications]

1. Evaluate each of the following definite integrals:

a. $\int_0^1 \frac{x}{\sqrt{x^2+1}}\,dx$ $= \sqrt{2} - 1$

b. $\int_1^2 x\sqrt{x^2+1}\,dx$ $= \frac{1}{3}\left(5\sqrt{5} - 2\sqrt{2}\right)$

c. $\int_{\pi/6}^{\pi/3} \sin^2\theta\,d\theta\,dx$ $= \frac{1}{12}\left(\pi - 3\sqrt{3} + 3\right)$

d. $\int_e^{e^2} \frac{1}{x}(\ln x)^2\,dx$ $= \frac{7}{3}$

2. Determine the area of the region enclosed in each case.

a. $y = \sin\theta$; $\theta = \frac{\pi}{6}$; $\theta = \frac{\pi}{3}$; $y = 0$. Area $= \frac{\sqrt{3}-1}{2}$ units of area

b. $y = \sin\theta$; $y = \cos\theta$; $\theta = 0$; $\theta = \frac{\pi}{2}$.
Area $= 2\sqrt{2} - 2$ units of area

c. $y = x^2 - 4x$; $y = x$.
Area $= \frac{125}{6}$ units of area

d. $y = x^2 - 4x$; $y = 6 - 3x^2$.
Area $= \frac{19\sqrt{7}}{6}$ units of area

e. $y = e^x$; $y = e^{-x}$; $x = -1$; $x = 1$.
Area $= 2e + \frac{2}{e}$ units of area