

THE UNIVERSITY OF CALGARY  
MATHEMATICS 249  
FINAL EXAMINATION, WINTER 2003  
TIME: 2 HOURS

NAME \_\_\_\_\_ ID \_\_\_\_\_

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Total (max. 75)	

SHOW ALL WORK. SIMPLIFY ALL ANSWERS AS MUCH AS POSSIBLE. NO CALCULATORS PLEASE.

THE MARKS FOR EACH PROBLEM ARE GIVEN TO THE LEFT OF THE PROBLEM NUMBER. TOTAL MARKS [75]. THIS EXAM HAS 9 PAGES INCLUDING THIS ONE.

[5] 1. Find and simplify  $\frac{d}{dx} \left( \frac{x}{x^3 - \ln x} \right)$ .

[5] 2. Find and simplify  $\frac{d}{dx} (e^{\sec^2 x})$ .

[5] 3. Find and simplify  $\frac{d}{dx} ((\tan 3x)\sqrt{1-x^2})$ .

[5] 4. Use logarithmic differentiation to find and simplify  $\frac{d}{dx} ((x+1)^{\sqrt{x}})$ .

[8] 5. (a) Use implicit differentiation to find and simplify  $\frac{dy}{dx}$  where  $e^x + \ln y = \cos x + \sin(\pi y)$ .

(b) Verify that the point  $(0, 1)$  lies on the graph of  $e^x + \ln y = \cos x + \sin(\pi y)$ .

(c) Find the equation of the tangent line to the graph of  $e^x + \ln y = \cos x + \sin(\pi y)$  at the point  $(0, 1)$ .

[5] 6. Prove the formula for  $\frac{d}{dx}(\cot x)$ . You may use the formulas for the derivatives of  $\sin x$  and  $\cos x$ .

[6] 7. USE THE DEFINITION OF DERIVATIVE to find  $\frac{d}{dx} \left( \frac{1}{4-x^2} \right)$ .

[10] 8. For the function  $f(x) = \frac{x^{1/3}}{x-2}$ ,

(a) show that  $f'(x) = -\frac{2x+2}{3x^{2/3}(x-2)^2}$ .

Then find (b) the domain of  $f(x)$ ; (c) the critical points; (d) the intervals of increase and decrease; (e) all local maxima and local minima.

[5] 9. For the function  $f(x) = xe^x$ , you are given that  $f'(x) = (x+1)e^x$  and  $f''(x) = (x+2)e^x$ . Find the intervals on which  $f(x)$  is concave up and where it is concave down. Then find all points of inflection.

[5] 10. Find all constants  $k$  so that the function

$$f(x) = \begin{cases} x^3 + 4x^2 + x + 7 & \text{if } x \leq k, \\ 7 - 3x & \text{if } x > k \end{cases}$$

is continuous at  $x = k$ . Also, for each such value of  $k$ , determine whether  $f(x)$  is differentiable at  $x = k$ .

[5] 11. Find and simplify  $\int \sqrt{\sin 4x} \cos 4x \, dx$ .

[5] 12. Find and simplify  $\int_0^1 (x^{1/3} - x^3) \, dx$ .



[6] 13. Do **ONE** of the following two problems.

(a) A ladder 4 metres long is leaning against a house (see Figure (a)). Its base starts to slide away from the house at  $1/2$  metre per second. How fast is the top of the ladder moving down the house when the top is 3 metres from the ground?

(b) A rectangle has its base on the  $x$  axis, its left side on the  $y$  axis, and its upper right-hand vertex on the curve  $y = 4/x^2$ . (See Figure (b).) What is the smallest **perimeter** the rectangle can have, and what are its dimensions?

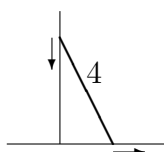


Figure (a)

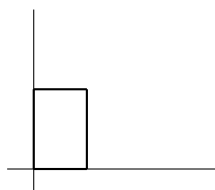


Figure (b)