The University of Calgary
Department of Mathematics and Statistics
MATH 249-01
Quiz # 2R

Fall,2006

Name:______________________________**I.D.#:**__________________

1. For $g(x) = \sqrt{4-x}$ and $f(x) = \dfrac{1}{2-x}$ find $f \circ g$ and its domain. [3]

2. For $\quad f(x) = \dfrac{1}{x+3}\left(1 - \dfrac{4}{1-x}\right) \quad$ find $\lim f(x)$ as

   (a) $x \to -3$
   (b) $x \to -\infty$
   (c) $x \to 1^+$ [4]

3. For $\quad g(x) = \dfrac{x^2-9}{\sqrt{3-x}} \quad$ find $\lim g(x)$ as

   (a) $x \to 3^+$
   (b) $x \to 3^-$ [3]

SOLUTION

For 1)

domains $\quad D_g = \{x \leq 4\} \quad D_f = \{x \neq 2\}$

$f \circ g(x) = f(g(x)) = \dfrac{1}{2-(g(x))} = \dfrac{1}{2-\sqrt{4-x}}$ we cannot have 0 in the denom

so $2 - \sqrt{4-x} \neq 0 \quad 2 \neq \sqrt{4-x} \quad 4 \neq x - 4 \quad x \neq 0$ and $x \leq 4$

together $D_{f \div g} = (-\infty, 0) \cup (0, 4]$

we can also simplify $f \circ g(x) = \dfrac{1}{2-\sqrt{4-x}} \cdot \dfrac{2+\sqrt{4-x}}{2+\sqrt{4-x}} = \dfrac{2+\sqrt{4-x}}{x}$

For 2a)

for $x \neq -3$ and $x \neq 1$

$.f(x) = \dfrac{1}{x+3}\left(1 - \dfrac{4}{1-x}\right) = \dfrac{1}{x+3}\left(\dfrac{1-x-4}{1-x}\right) = \dfrac{-(x+3)}{(x+3)(x-1)} = \dfrac{-1}{1-x} = \dfrac{1}{x-1}$

so for a) $\quad \lim\limits_{x \to -3^-} f(x) = \dfrac{-1}{4}$

for b) $\quad \lim\limits_{x \to -\infty} \dfrac{1}{x+3}\left(1 - \dfrac{4}{1-x}\right) = \text{"}\dfrac{1}{-\infty}\text{"}\left(1 - \dfrac{4}{+\infty}\right) = 0 \cdot 1 = 0$

or from the simplified from $\lim_{x\to-\infty}\frac{-1}{1-x} = "\frac{-1}{\infty}" = 0$

for c) $\lim_{x\to1^{+-}} f(x) = \lim_{x\to1^{+-}}\frac{1}{x-1} = "\frac{1}{0^{+}}" = +\infty$

For 3)

the function $g(x) = \frac{x^2-9}{\sqrt{3-x}}$ is defined only for $x < 3$

in **a**) **limit** is DNE. does not exist since no values for $x > 3$

in **b**)the limit is "$\frac{0}{0}$"so simplify: $g(x) = \frac{x^2-9}{\sqrt{3-x}} = \frac{(x-3)(x+3)}{\sqrt{3-x}} = \frac{-(3-x)(x+3)}{\sqrt{3-x}} =$
$-\sqrt{3-x}\,(x+3)$

and the limit is 0.