The University of Calgary
Department of Mathematics and Statistics
MATH 249-01
Quiz #3R

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Name:______________________________**I.D.#:**__________________

1. Using the definition of derivative find $f'(-1)$ if $f(x) = \sqrt{12+3x}$. [3]
2. Using Differentiation Rules find f' if $f(x) = \dfrac{x}{\sqrt{1-2x}}$ in the domain. [3]
3. Find an equation of the tangent to $y = \left(\dfrac{x^2}{2} + 3x\right)\left(1 - \dfrac{6}{x+1}\right)$ at $x = 2$. [4]

Solution

For 1)

first $\quad f(-1) = \sqrt{12+3x} = 3$ then

$$f'(-1) = \lim_{x\to -1} \frac{f(x) - f(-1)}{x-(-1)} = \lim_{x\to -1} \frac{\sqrt{12+3x} - 3}{x+1} = \lim_{x\to -1} \frac{\sqrt{12+3x} - 3}{x+1} \cdot \frac{\sqrt{12+3x} + 3}{\sqrt{12+3x} + 3}$$

$$= \lim_{x\to -1} \frac{(12+3x) - 9}{(x+1)\left(3 + \sqrt{12+3x}\right)} =$$

$$= \lim_{x\to -1} \frac{3(x+1)}{(x+1)\left(3 + \sqrt{12+3x}\right)} = \lim_{x\to -1} \frac{3}{\left(3 + \sqrt{12+3x}\right)} = \frac{3}{6} = \frac{1}{2}$$

check by Chain Rule:

$$f'(x) = \left[(12+3x)^{\frac{1}{2}}\right]' = \left(\frac{1}{2}\right)(12+3x)^{-\frac{1}{2}} \cdot (3) \text{ and at } x = 1 \text{ we get } \frac{3}{2\cdot 3} = \frac{1}{2}$$

For 2)

use Quotient and Chain Rules for $x < \frac{1}{2} \qquad (1 - 2x > 0)$

$$f'(x) = \frac{(x)'\sqrt{1-2x} - x\left(\sqrt{1-2x}\right)'}{\left(\sqrt{1-2x}\right)^2} = \frac{\sqrt{1-2x} - x \cdot \dfrac{-2}{2\sqrt{1-2x}}}{1-2x}$$

we can simplify

$$f'(x) = \frac{1 - 2x + x}{(1-2x)\sqrt{1-2x}} = \frac{1-x}{(1-2x)^{\frac{3}{2}}}$$

For 3)

$$y = \left(\frac{x^2}{2} + 3x\right)\left(1 - \frac{6}{x+1}\right) = -8 \text{ at } x = 2 \qquad P\,(2, -8)$$

so an equation of the tangent is $\qquad y = m_t\,(x-2) - 8$

to find the slope use Product Rule $y' = \left[\left(\frac{x^2}{2}+3x\right)\left(1-\frac{6}{x+1}\right)\right]' =$

$= \left(\frac{x^2}{2}+3x\right)'\left(1-\frac{6}{x+1}\right) + \left(\frac{x^2}{2}+3x\right)\left[1-6\,(x+1)^{-1}\right]' =$

$\left(\frac{1}{2}\cdot 2x+3\right)\left(1-\frac{6}{x+1}\right) + \left(\frac{x^2}{2}+3x\right)(-6)\,(-1)(x+1)^{-2} \,=\, (x+3)\left(1-\frac{6}{x+1}\right) + \frac{3x^2+18x}{(x+1)^2}$

at $x=2$ $\quad m_t = -5 + \frac{48}{9} = \frac{1}{3}$ $\quad$ so $y = \frac{1}{3}(x-2) - 8$ $\quad$ OR $\quad x - 3y = 26.$