The University of Calgary
Department of Mathematics and Statistics
MATH 249-01
Quiz #3W

FALL 2006

Name:______________________________**I.D.#:**__________________

1. Using the definition of derivative find $f'(2)$ if $f(x) = \dfrac{x}{5-x}$. [3]

2. Find y' if $y = (\dfrac{x^3}{3} - 2\sqrt{x+2})(4 + \dfrac{1}{\sqrt{4x}})$ for $x > 0$. [3]

3. Find an equation of the tangent line to $y = \dfrac{\sqrt[3]{2-3x}}{x}$ at $x = -2$. [4]

Solution
For 1)

$$f'(2) = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h} = \lim_{h\to 0}\frac{\dfrac{2+h}{5-(2+h)} - \dfrac{2}{3}}{h} =$$

$$= \lim_{h\to 0}\frac{\dfrac{2+h}{3-h} - \dfrac{2}{3}}{h} = \lim_{h\to 0}\frac{1}{h}\left[\frac{6+3h-6+2h}{3(3-h)}\right] =$$

$$= \lim_{h\to 0}\frac{1}{h}\cdot\frac{5h}{3(3-h)} = \lim_{h\to 0}\frac{5}{3(3-h)} = \frac{5}{9}$$

(Check by rules $f'(x) = \dfrac{5-x-x(-1)}{(5-x)^2} = \dfrac{5}{(5-x)^2}$ at $x = 2$ $f'(2) = \frac{5}{9}$)

also $$f'(2) = \lim_{x\to 2}\frac{f(x)-f(2)}{x-2} = \lim_{x\to 2}\frac{\dfrac{x}{5-x} - \dfrac{2}{3}}{x-2} = \lim\frac{\frac{3x-10+2x}{3(5-x)}}{x-2} =$$

$$= \lim_{x\to 2}\frac{5(x-2)}{3\,(x-2)\,(5-x)} = \lim_{x\to 2}\frac{5}{3(5-x)} = \frac{5}{9}$$

For 2)

use Product Rule $$y' = \left[(\frac{x^3}{3} - 2\sqrt{x+2})(4 + \frac{1}{\sqrt{4x}})\right]' =$$

$$= \left[\frac{x^3}{3} - 2\,(x+2)^{\frac{1}{2}}\right]'(4 + \frac{1}{\sqrt{4x}}) + (\frac{x^3}{3} - 2\sqrt{x+2})(4 + \frac{1}{2}x^{-\frac{1}{2}})\ ,$$

now Power Rule

$$y' = \left[\frac{1}{3}\cdot 3x^2 - 2\cdot\frac{1}{2}\,(x+2)^{-\frac{1}{2}}\right](4 + \frac{1}{\sqrt{4x}}) + (\frac{x^3}{3} - 2\sqrt{x+2})(0 + \frac{1}{2}\left(\frac{-1}{2}\right)x^{-\frac{3}{2}})$$

so $$y' = \left[x^2 - (x+2)^{-\frac{1}{2}}\right](4 + \frac{1}{\sqrt{4x}}) + (\frac{x^3}{3} - 2\sqrt{x+2})\left(\frac{-1}{4}\right)x^{-\frac{3}{2}} \qquad \text{for } x > 0$$

For 3)

$y = \dfrac{\sqrt[3]{2-3x}}{x} = \dfrac{\sqrt[3]{8}}{-2} = -1$ at $x = -2$ so the point is $P\,(-2,-1)$

st. line through P $\quad y = -1 + m(x+2)$

slope of the tangent is given by $y' = \left(\dfrac{\sqrt[3]{2-3x}}{x}\right)'$ at $x = -2$

we can use Quotient and Chain Rules

$$y' = \frac{\left[(2-3x)^{\frac{1}{3}}\right]' x - \sqrt[3]{2-3x}\,(x)'}{x^2} = \frac{\frac{1}{3}(2-3x)^{\frac{-2}{3}}(-3x) - \sqrt[3]{2-3x}}{x^2} = \frac{\frac{-x}{(2-3x)^{\frac{2}{3}}} - \sqrt[3]{2-3x}}{x^2}$$

so $m = \dfrac{\frac{2}{(8^{\frac{1}{3}})^2} - \sqrt[3]{8}}{4} = \dfrac{\frac{1}{2} - 2}{4} = \dfrac{\frac{-3}{2}}{4} = -\dfrac{3}{8}$ $\qquad$ and $y = -1 - \frac{3}{8}(x+2)$ or $8y + 3x + 14 = 0$