The University of Calgary
Department of Mathematics and Statistics
MATH 249-01
Quiz # 4R

FALL 2006

Name:______________________________**I.D.#:**__________________

1. Find an equation of the tangent to

$$\sqrt{2x-y} = \frac{3x}{y} - 3$$

at the point $(6,3)$. [4]

2. Find the seconf derivative of $f(x) = \sqrt{4+x^2}$.Simplify. [3]

3. Find a general antiderivative of $f(x) = \sqrt{x}(2x - \frac{3}{\sqrt{x}}) + \cos(\frac{1-x}{3})$ for $x > 0$. [3]

Solution

For 1)

an equation $\quad y = m\,(x-6) + 3$ and to find m use

implicit differentiation $\quad \left[(2x-y)^{\frac{1}{2}}\right]' = 3\left(\frac{x}{y}\right)' - 0$

$\frac{1}{2}\,(2x-y)^{-\frac{1}{2}} \cdot (2x-y)' = 3 \cdot \frac{y - xy'}{y^2}$ multiply by 2

$(2x-y)^{-\frac{1}{2}}\,(2-y') = \frac{6\,(y-xy')}{y^2}$

now,$x=6, y=3, y'=m$

$\frac{1}{\sqrt{9}}\,(2-m) = \frac{6(3-6m)}{3^2} = \frac{6-12m}{3}$ so $2-m = 6-12m$

and $11m = 4 \quad y = \frac{4}{11}\,(x-6) + 3$ OR $11y - 4x = 9$

For 2)

by Chain rule and then Quotient rule

$f'(x) = \left[(4+x^2)^{\frac{1}{2}}\right]' = \frac{1}{2}\,(4+x^2)^{-\frac{1}{2}}\,2x = x\,(4+x^2)^{-\frac{1}{2}} = \frac{x}{\sqrt{4+x^2}}$

$f''(x) = \frac{\sqrt{4+x^2} - x \cdot \frac{2x}{2\sqrt{4+x^2}}}{4+x^2} = \frac{\frac{4+x^2-x^2}{\sqrt{4+x^2}}}{4+x^2} = \frac{4}{(4+x^2)^{\frac{3}{2}}}$

or by Product and Chain rules

$f''(x) = \left[x\,(4+x^2)^{-\frac{1}{2}}\right]' = (4+x^2)^{-\frac{1}{2}} - \frac{1}{2}x\,(4+x^2)^{-\frac{3}{2}}\,2x = \frac{4+x^2-x^2}{(4+x^2)^{\frac{3}{2}}} = \frac{4}{(4+x^2)^{\frac{3}{2}}}$

For 3)

$\int \sqrt{x}(2x - \frac{3}{\sqrt{x}}) + \cos(\frac{1-x}{3})dx = \int (2x\sqrt{x} - 3)dx + \int \cos(\frac{1}{3} - \frac{1}{3}x)dx =$

$$= 2\int x^{\frac{3}{2}} dx - 3\int dx + \frac{\sin\left(\dfrac{1}{3} - \dfrac{1}{3}x\right)}{\dfrac{-1}{3}} = \tfrac{4}{5}x^{\frac{5}{2}} - 3x - 3\sin\left(\frac{1-x}{3}\right) + c$$

using $\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b) + c$