The University of Calgary
Department of Mathematics and Statistics
MATH 249//01
Quiz # 4W

FALL 2006

Name:____________________________**I.D.#:**__________________

1. Find an equation of the tangent to
$$\cos(\pi xy) + \pi x^2 y = -\frac{\pi}{4}$$
at the point $(\frac{1}{2}, -1)$. [4]

2. Find the second derivative of $\quad f(x) = \dfrac{x}{1+x^2}$. Simplify. [3]

3. Find a general antiderivative of $f(x) = \dfrac{8\sqrt{x} - x^3 + 2}{2x^2}$ for $x > 0$. [3]

Solution **For1)**

an equation is $\quad y = m(x - \frac{1}{2}) - 1$ for m use the implicit differentiation

$-\sin(\pi xy)\pi\,(xy)' + \pi\,(x^2y)' = 0 \qquad$ product and chain rules

$-\sin(\pi xy)\pi\,(y + xy') + \pi\,(2xy + x^2y') = 0$ cancel π

and substitute $x = \frac{1}{2}, y = -1, y' = m, \sin(-\frac{\pi}{2}) = -1$

$-1 + \frac{1}{2}m - 1 + \frac{1}{4}m = 0 \qquad \frac{3}{4}m = 2$

so $\quad m = \dfrac{8}{3}$ and an equation is $y = \frac{8}{3}(x - \frac{1}{2}) - 1$ or $3y - 8x = -7$

For 2)

by Quotient Rule

$$f'(x) = \left(\frac{x}{1+x^2}\right)' = \frac{1 + x^2 - 2x^2}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$$

$$f''(x) = \left(\frac{1-x^2}{(1+x^2)^2}\right)' = \frac{-2x\,(1+x^2)^2 - (1-x^2)\,2(1+x^2)2x}{(1+x^2)^4} =$$

$$= \frac{-2x\,(1+x^2)\,[1 + x^2 + 2 - 2x^2]}{(1+x^2)^4} = \frac{-2x\,[3 - x^2]}{(1+x^2)^3}$$

OR

$$f'(x) = \left[x\,(1+x^2)^{-1}\right]' = (1+x^2)^{-1} - x\,(1+x^2)^{-2}\,2x = (1+x^2)^{-1} - 2x^2\,(1+x^2)^{-2}$$

$$f''(x) = -2x(1+x^2)^{-2} - 4x(1+x^2)^{-2} - 2x^2(-2)(1+x^2)^{-3}2x$$

$$= -6x(1+x^2)^{-2} + 8x^3\,(1+x^2)^{-3}.$$

For 3)

$$\int \frac{8\sqrt{x} - x^3 + 2}{2x^2}dx = 4\int \frac{\sqrt{x}}{x^2}dx - \tfrac{1}{2}\int \frac{x^3}{x^2}dx + \int \frac{1}{x^2}dx =$$

$$= 4\int x^{-\frac{3}{2}}dx - \tfrac{1}{2}\int x dx + \int x^{-2}dx = -8x^{-\frac{1}{2}} - \frac{1}{4}x^2 - x^{-1} + c, x > 0.$$