

The University of Calgary
 Department of Mathematics and Statistics
 MATH 249- 01
 Quiz #5R

Fall 2006

Name: _____ I.D.#: _____

1. Solve for x : $2 \ln(x + 1) - \ln(4x) = 0$. [3]

2. For $f(x) = 2^{\sqrt{x}} + (\frac{1}{x})^x$ find the domain and $f'(x)$. [4]

3. How long does it take to double your investment if the annual interest is 2.5% compounded monthly? [3]

SOLUTION

For 1)

simplify first $\ln(x + 1)^2 = \ln(4x)$ then apply exp. f.

$$(x + 1)^2 = 4x \quad x^2 - 2x + 1 = 4x$$

$$x^2 - 2x + 1 = 0 \quad (x - 1)^2 = 0$$

$x = 1$ is the solution.

For 2)

it must $x > 0$ rewrite $(\frac{1}{x})^x = e^{x \ln \frac{1}{x}} = e^{-x \ln x}$

$$f'(x) = (2^{\sqrt{x}})' + (e^{-x \ln x})' = 2^{\sqrt{x}} \ln 2 (\sqrt{x})' + e^{-x \ln x} (-x \ln x)' =$$

$$= \frac{\ln 2}{2\sqrt{x}} 2^{\sqrt{x}} - e^{-x \ln x} (\ln x + 1)$$

or log.diff BUT only for $u = (\frac{1}{x})^x \quad \ln u = \ln (\frac{1}{x})^x = -x \ln x$

$$\text{then } \frac{u'}{u} = -\ln x - x \cdot \frac{1}{x} \quad u' = u(-1)(\ln x + 1) = -(\frac{1}{x})^x (\ln x + 1)$$

For 2)

the correct formula for the amount of money is $A(t) = A_0 \left(1 + \frac{p}{100n}\right)^{nt}$

t in years ,the initial amount A_0 should change to $2A_0, p = 2.5, n = 12$

$$\text{thus } 2A_0 = A_0 \left(1 + \frac{2.5}{1200}\right)^{12t} \quad 2 = \left(\frac{12025}{12000}\right)^{12t} \text{ solve for } t$$

$$\ln 2 = 12t \ln \frac{12025}{12000} \quad t = \frac{\ln 2}{12 \ln \frac{12025}{12000}} = 27.755 \text{ years}$$