

The University of Calgary
 Department of Mathematics and Statistics
 MATH 249- 01
 Quiz #5W

Fall 2006

Name: _____ I.D.#: _____

1. Solve for x : $\frac{1}{3^{2x+1}} = \frac{5}{9^{3x}}$. [3]

2. For $f(x) = x^{\sqrt{x}} + \ln(2x - 1)$ find the domain and $f'(x)$. [4]

3. After 3 days a sample of radon decayed to 45 kg. After one more day to 35 kg.
 What was the original amount? [3]

SOLUTION

For 1)

apply \ln to both sides $\frac{1}{3^{2x+1}} = \frac{5}{9^{3x}}$

$\ln 1 - (2x + 1) \ln 3 = \ln 5 - 3x \ln 9$

$3x \ln 3^2 - 2x \ln 3 = (6x - 2x) \ln 3 = \ln 5 + \ln 3 = \ln 15$

$4x \ln 3 = \ln 15$

or $x(3 \ln 9 - 2 \ln 3) = x \ln \frac{3^6}{3^2}$

and finally $x \ln 3^4 = \ln 15$ $x = \frac{\ln 15}{\ln 81} = \frac{\ln 15}{4 \ln 3}$

OR cross multiply first $\frac{9^{3x}}{3^{2x+1}} = \frac{3^{6x}}{3^{2x+1}} = 3^{4x-1} = 5$ $3^{4x} = 15$

and then log.f. $4x = \log_3 15$ $x = \frac{1}{4} \log_3 15$

For 2)

it must $x > 0$ and $2x - 1 > 0$ so $x > \frac{1}{2}$ rewrite $x^{\sqrt{x}} = e^{\sqrt{x} \ln x}$

$f'(x) = (e^{\sqrt{x} \ln x})' + (\ln(2x - 1))' = e^{\sqrt{x} \ln x} (\sqrt{x} \ln x)' + \frac{1}{2x - 1} \cdot 2 =$

$= e^{\sqrt{x} \ln x} \left(\frac{1}{2\sqrt{x}} \cdot \ln x + \frac{\sqrt{x}}{x} \right) + \frac{2}{2x - 1} = e^{\sqrt{x} \ln x} \left(\frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}} \right) + \frac{2}{2x - 1}$

or log.diff BUT only for $u = x^{\sqrt{x}}$ $\ln u = \ln x^{\sqrt{x}} = \sqrt{x} \ln x$

then $\frac{u'}{u} = \frac{\ln x}{2\sqrt{x}} + \frac{x}{\sqrt{x}}$ $u' = u \left(\frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}} \right) = x^{\sqrt{x}} \left(\frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}} \right)$

For 2)

the correct formula for the amount of radon $A(t) = A_0 e^{kt}$ where $k < 0$

t in days ,the initial amount $A_0 = ?$

given if $t = 3$ $45 = A_0 e^{3k}$ if $t = 4$ $35 = A_0 e^{4k}$

so $A_0 = \frac{45}{e^{3k}} = \frac{35}{e^{4k}}$ first solve for k $e^{4k-3k} = \frac{35}{45}$ $e^k = \frac{7}{9}$

$k = \ln \frac{7}{9} = -0.2513144$ and $A_0 = \frac{45}{e^{3k}} = 45 \cdot \frac{9^3}{7^3} = 95.64kg$