MATH 249- 01

Midterm 55 minutes

Fall 2006

Name:	**I.D.#**

1. Find $\lim\limits_{x\to\infty} \dfrac{\cos(x^2+2)}{2-x^2}$. [5]

2. If $\sin\theta = -\frac{3}{7}$and $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$ find $\csc 2\theta$.**NO CALCULATORS**! [5]

3. Find the derivative of $f(x) = \dfrac{\cos\sqrt{x}}{\sqrt{x}}$ for $x > 0.$ Simplify. [5]

4. Does the graph of $y = x - \cos x$ cross the $x-$ axis?
   i.e.Is there any x for which the value $y = 0$? Explain.State the Theorem used. [5]

5. Find the values of a and b so that the function f is continuous everywhere

$$f(x) = \begin{cases} \cos^2\dfrac{\pi}{x} & \text{for} \quad x < -4 \\ ax+b & \text{for} \quad -4 \le x \le 3 \\ \dfrac{6\sin(x-3)}{x^2-9} & \text{for} \quad x > 3 \end{cases}.$$ [10]

6. Sketch the graph of ONE function satisfying all the following conditions:

   (a) f is defined on$(-\infty, 2]$

   (b) f is discontinuous at $x = 0, 1$ where $\lim\limits_{x\to 0^-} f(x) = f(0) = 3$, $\lim\limits_{x\to 1} f(x)$ DNE(does not exist),otherwise continuous

   (c) $x = 0$ is a vertical asymptote and $y = 1$ is a horizontal asymptote

   (d) f is not differentiable at $x = -1, 0, 1$ (no $f'(-1)$) otherwise differentiable and $f'(x) = 0$ for all $x \in (1, 2)$, also $f'(-2) = 0$. [10]

SOLUTION

For 1)

$\lim\limits_{x\to\infty} \dfrac{\cos(x^2+2)}{2-x^2} = 0$ by Squ.Theorem

since $\quad -1 \le \cos(x^2+2) \le 1 \qquad \dfrac{-1}{2-x^2} \ge \dfrac{\cos(x^2+2)}{2-x^2} \ge \dfrac{1}{2-x^2}$

(denom. is negative)

$$\text{and} \qquad \lim_{x\to\infty} \frac{\pm 1}{2-x^2} = 0.$$

For 2)

since $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$ means that $\cos\theta = -\sqrt{1 - \left(-\frac{3}{7}\right)^2} = -\sqrt{\dfrac{40}{49}} = \dfrac{-2\sqrt{10}}{7}$

then $\csc 2\theta = \dfrac{1}{\sin 2\theta} = \dfrac{1}{2\sin\theta\cos\theta} = \dfrac{1}{2} \cdot \dfrac{-7}{3} \cdot \dfrac{-7}{2\sqrt{10}} = \dfrac{49}{120}\sqrt{10}$

.

For 3)

for $x > 0$ by Quot.and Chain rules

$$f'(x) = \left[\frac{\cos\sqrt{x}}{\sqrt{x}}\right]' = \frac{(\cos\sqrt{x})'\sqrt{x} - \cos\sqrt{x}\left(\sqrt{x}'\right)}{\left(\sqrt{x}\right)^2} = \frac{-\sin\sqrt{x}\cdot\frac{1}{2\sqrt{x}}\cdot\sqrt{x} - \cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}}}{x}$$

$$= \frac{-\sqrt{x}\sin\sqrt{x} - \cos\sqrt{x}}{2x\sqrt{x}}$$

OR by Product Rule

$$f'(x) = \left(x^{-\frac{1}{2}}\cos\sqrt{x}\right)' = \left(x^{-\frac{1}{2}}\right)'\cos\sqrt{x} + x^{-\frac{1}{2}}(\cos\sqrt{x})' =$$

$$= -\tfrac{1}{2}x^{-\frac{3}{2}}\cos\sqrt{x} + x^{-\frac{1}{2}}(-\sin\sqrt{x})\left(-\tfrac{1}{2}x^{-\frac{1}{2}}\right) = -\frac{\cos\sqrt{x} + \sqrt{x}\sin\sqrt{x}}{2x\sqrt{x}}$$

For 4)

the function $f(x) = x - \cos x$ is continuous everywhere

so check some values : $f(0) = -1 < 0$

$f(\frac{\pi}{2}) = \dfrac{\pi}{2} > 0$ since $\cos\frac{\pi}{2} = 0$

one value negative ,one positive thus there must be $c \in \left(0, \frac{\pi}{2}\right)$

such that $f(c) = 0$ $\quad$ c is the $x-$intercept (zero)

By IVT

For 5)

for any values a and b the function f is defined everywhere and it is continuous there except at $x = -4$ and $x = 3$

at $x = -4$ $\quad$ $f(-4) = -4a + b = \lim\limits_{x\to -4^+} f(x)$ and

$$\lim_{x\to -4^-} f(x) = \lim_{x\to -4^-} \cos^2\frac{\pi}{x} = \left[\cos\left(-\frac{\pi}{4}\right)\right]^2 = \frac{1}{2}$$

so it must $\quad$ -4$a + b = \frac{1}{2}$

at $x = 3$ $\quad$ $f(3) = 3a + b = \lim\limits_{x\to 3^-} f(x)$ and

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \frac{6\sin(x-3)}{x^2-9} = 6\lim_{x\to 3^+} \frac{\sin(x-3)}{x-3}\cdot\frac{1}{x+3} = 6\cdot 1\cdot\frac{1}{6} = 1$$

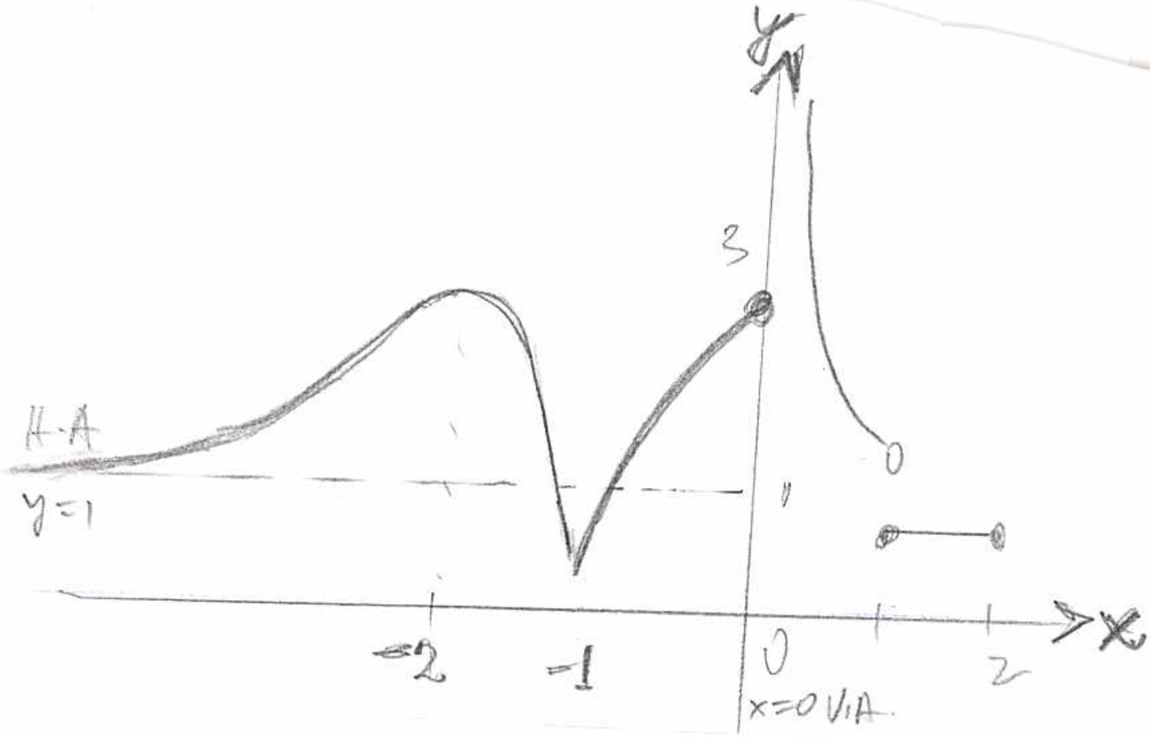
using $\lim\limits_{h\to 0}\dfrac{\sin h}{h} = 1$ so $\quad$ $3a + b = 1$

now,solve the system $\quad$ $-4a + b = \frac{1}{2}$ $\quad$ $3a + b = 1$

subtract the equations to get $\quad$ $7a = 1 - \frac{1}{2} = \frac{1}{2}$

so $a = \dfrac{1}{14}$ and $b = 1 - 3a = \dfrac{11}{14}$.

For 6)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x
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