MATH 251/249
Worksheet #1 .

For 1 a) $|2x+1| \leq |x-2|$

Since $|...|$ is always positive or zero we can square both sides and the sign of the inequality stays the same: $(2x+1)^2 \leq (x-2)^2$ since $|...|^2 = (...)^2$ Now

$4x^2+4x+1 \leq x^2-4x+4$,everything on one side: $3x^2+8x-3 \leq 0$

$(3x-1)(x+3) \leq 0$,thus split points are: $x=-3, \frac{1}{3}$,

testing: $--^{pos}---_{-3}--^{neg}--_{\frac{1}{3}}--^{pos}---$

so the solutions set is the closed interval $\left[-3, \frac{1}{3}\right]$

b) $\frac{3}{x+1} > \frac{1}{3}$ for $x \neq -1$

everything on one side and common denominator: $\frac{3\cdot 3-(x+1)}{(x+1)3} > 0$,simplify:

$\frac{9-x-1}{3(x+1)} > 0$ then $\frac{8-x}{(x+1)(3)} > 0$.So split points are : $x = 8, -1$

testing: $--^{neg}---_{-1}--^{pos}--_{8}--^{neg}---$

solution set is the open interval $]-1,8[$ or $(-1,8)$.

For 2) $x^2+4x+y^2-2y=11$ $x^2+4x+4+y^2-2y+1 = 11+4+1$

Complete the squares: $(x+2)^2+(y-1)^2 = 16$ so $(x+2)^2+(y-1)^2 = 4^2$

thus $r=4$ and the point $C(-2,1)$ is the centre.

For 3a) $|x+1|+2>0$

Since $|...|$ is always positive or zero $|x+1|+2$ is always positive ,so solution set $]-\infty,+\infty[$

b) $\frac{3}{x+1} \geq \frac{2}{x+3}$ for $x \neq -1, -3$

everything on one side and common denominator: $\frac{3(x+3)-2(x+1)}{(x+1)(x+3)} \geq 0$,simplify:

$\frac{3x+9-2x-2}{(x+1)(x+3)} \geq 0$ then $\frac{(x+7)}{(x+1)(x+3)} \geq 0$.So split points are : $x=-7,-3,-1$

testing: $-^{neg}-_{-7}---^{pos}---_{-3}--^{neg}--_{-1}--^{pos}---$

solution set: $[-7,-3[\cup]-1,+\infty[$ or $[-7,-3)\cup(-1,+\infty)$.

For 4) For $l_1: 3x+2y=1$ $l_2: 2y-3x=0$ $l_3: 3x-2y=0$ and $l_4: 2x-3y=2$

find the slopes: $m_1 = -\frac{3}{2}$, $m_2 = \frac{3}{2}, m_3 = \frac{3}{2}, m_4 = \frac{2}{3}$

so $l_2 \|\ l_3$ since they have the same slope

and $l_1 \perp l_4$ since $m_1 \cdot m_4 = -1$.

For 5 a) $\frac{1}{x+1} \leq 1+x$ for $x \neq -1$

everything on one side and common denominator: $\frac{1-(x+1)^2}{(x+1)} \leq 0$,simplify:

$\frac{1-x^2-2x-1}{(x+1)} \leq 0$ then $\frac{-x(x+2)}{(x+1)} \leq 0$.So split points are : $x=0,-2,-1$

testing $-^{pos}--_{-2}--^{neg}-_{-1}--^{pos}---_{0}--^{neg}--$

solution set: $[-2,-1[\cup[0,+\infty[$ or $[-2,-1)\cup[0,+\infty)$.

b) $|3x-2|>0$

Since $|...|$ is always positive or zero we have to elliminate zero :$3x-2=0$ for $x=\frac{2}{3}$
The solutions : $x\neq\frac{2}{3}$ or $\left]-\infty,\frac{2}{3}\right[\cup\left]\frac{2}{3},+\infty\right[$ or $\left(-\infty,\frac{2}{3}\right)\cup\left(\frac{2}{3},+\infty\right)$

For 6)

$\perp$ to x-axis means a vertical line so $x=-1$ (y is any).

line parallel to the x-axis means a horizontal line so $y=3$ (x any)

For 7 a) $3x+7>x^2$

Everything on one side: $0>x^2-3x-7$ now find the roots ,firs discriminant $D=(-3)^2-4\cdot1\cdot(-7)=9+28=37$, so using the formula roots are $x_1=\frac{3-\sqrt{37}}{2}\doteq-1.54$ and $x_2=\frac{3+\sqrt{37}}{2}\doteq4.54$

Now testing : $--^{pos}--_{x_1}--^{neg}---_{x_2}--^{pos}-$

OR parabola open up and it is below the x-axis if $x\in\left]-1.54,4.54\right[$.
(between the roots $x\in(x_1,x_2)$)

b) $\dfrac{x}{2}<\dfrac{2}{x+3}$. for $x\neq-3$

everything on one side and common denominator: $\dfrac{x(x+3)-2\cdot2}{2(x+3)}<0$,simplify:

$\dfrac{x^2+3x-4}{2(x+3)}<0$ then $\dfrac{(x+4)(x-1)}{2(x+3)}<0$.So split points are : $x=-4,-3,1$

testing:$-^{neg}--_{-4}--^{pos}--_{-3}-^{neg}--_{1}-^{pos}--$

solution set: $\left]-\infty,-4\right[\cup\left]-3,1\right[$ or $(-\infty,-4)\cup(-3,-1)$.

For 8) $x^2-6x+y^2=7$ $x^2+y^2+2y=15$

Complete the squares : $x^2-6x+9+y^2=7+9$,$x^2+y^2+2y+1=1+15$ SO the equations are:

$(x-3)^2+y^2=16, x^2+(y+1)^2=16$ thus radii are the same $r=4$,
the centres are points $(3,0)$ and $(0,-1)$.

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For 9) for $h\neq0,7$ find common denominator first

$$\frac{\frac{3h+4}{7-h}-\frac{4}{7}}{\frac{25h}{7}}=\frac{\frac{7(3h+4)-4(7-h)}{7(7-h)}}{\frac{25h}{7}}=\frac{21h+28-28+4h}{7(7-h)}\cdot\frac{7}{25h}=\frac{1}{7-h}.$$

For 10) for $x\neq-1$

$$\frac{1}{1+\frac{1}{x+1}}=\frac{1}{\frac{x+1+1}{x+1}}=\frac{x+1}{x+2}\text{ and for }x\neq-2.$$

For 11) factor out the polynomials

$$\frac{x^3+5x^2+6x}{12+x-x^2}=\frac{x(x^2+5x+6)}{-(x^2-x-12)}=\frac{x(x+3)(x+2)}{-(x-4)(x+3)}=\frac{x(x+2)}{-(x-4)}$$

for $x\neq-3,4$.

For 12) factor out the polynomials ,then common denom.

$$\frac{x}{x^2+x-2}-\frac{2}{x^2-5x+4}=\frac{x}{(x+2)(x-1)}-\frac{2}{(x-4)(x-1)}=\frac{x(x-4)-2(x+2)}{(x+2)(x-1)(x-4)}=$$

$$=\frac{x^2-6x-4}{(x+2)(x-1)(x-4)}\text{ for }x\neq-2,1,4.$$

For 13)

$$\frac{x}{x-1}<\frac{1}{x+1}\qquad\text{for }x\neq\pm1$$

everything on one side and common denominator: $\frac{x(x+1)-(x-1)}{(x-1)(x+1)} < 0$

simplify $\frac{x^2+1}{(x-1)(x+1)} < 0$ the top has NO real roots,always positive

thus only two split points $x = \pm 1$

Now testing : $--^{pos}--_{-1}--^{neg}---_{1}--^{pos}-$

the solution set is $(-1,1)$.

For 14)

$\frac{x}{x-1} > \frac{4}{x}$ for $x \neq 1,0$

everything on one side and common denominator: $\frac{x^2-4(x-1)}{(x-1)(x)} > 0$

simplify $\frac{x^2-4x+4}{x(x-1)} > 0$ $\frac{(x-2)^2}{x(x-1)} > 0$

the top has a double root, the split points $x = 0,1,2$

but only $x = 0,1$ are switch points

Now testing : $--^{pos}--_{0}--^{neg}---_{1}--^{pos}-_{2}-^{pos}---$

check the split points!!

the solution set is $(-\infty,0)\cup(1,2)\cup(2,+\infty)$.