THE UNIVERSITY OF CALGARY
MATHEMATICS 249
FINAL EXAMINATION, FALL 2002
TIME: 2 HOURS

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Total (max. 75)	

SHOW ALL WORK. SIMPLIFY ALL ANSWERS AS MUCH AS POSSIBLE. NO CALCULATORS PLEASE.

THE MARKS FOR EACH PROBLEM ARE GIVEN TO THE LEFT OF THE PROBLEM NUMBER. TOTAL MARKS [75]. THIS EXAM HAS 9 PAGES INCLUDING THIS ONE.

[5] 1. Find and simplify $\lim_{x \to -5} \left(\dfrac{25 - x^2}{2x^2 + 3x - 35} \right)$.

[5] 2. Find and simplify $\dfrac{d}{dx} \ln(x^3 - \cos x)$.

[5] 3. Find and simplify $\dfrac{d}{dx}\Big(x\tan(e^{5x})\Big)$.

[5] 4. Find and simplify $\dfrac{d}{dx}\left(\dfrac{(1-x)^{1/3}}{3-\sqrt{x}}\right)$.

[6] 5. Do **ONE** of the following two problems.

(a) Use implicit differentiation to find and simplify $\dfrac{dy}{dx}$ where $\sin(5x - y) + 2x = y^2 + 7$.

(b) Use logarithmic differentiation to find and simplify $\dfrac{d}{dx}\left((\sec x)^{4x}\right)$.

[6] 6. Find and simplify the equation of the tangent line to the curve $y = \dfrac{8}{(x-1)^2}$ at the point where $x = 3$.

[6] 7. USE THE DEFINITION OF DERIVATIVE to find $\dfrac{d}{dx}\left(\sqrt{4-3x}\right)$.

[10] 8. You are given the function $f(x) = x - 4\sqrt{x}$, and that

$$f'(x) = 1 - \frac{2}{\sqrt{x}} \ , \qquad f''(x) = \frac{1}{x^{3/2}} \ .$$

For the function $f(x)$, find: (a) the domain of f; (b) the critical points; (c) the intervals of increase and decrease; (d) the intervals of concave up and concave down; (e) all local maxima and local minima. Also find (f) the absolute maximum and absolute minimum of $f(x)$ on the interval $[0, 25]$.

[6] 9. Find constants k and ℓ so that the function

$$f(x) = \begin{cases} k - 2x & \text{if } x \leq 2, \\ \ell\sqrt{x+7} & \text{if } x > 2 \end{cases}$$

is both continuous and differentiable at $x = 2$.

[5] 10. Find and simplify $\displaystyle\int \frac{x^2}{(x^3-8)^3}\, dx$.

[5] 11. Find and simplify $\int e^{-x}\cos(e^{-x})\,dx$.

[5] 12. Find and simplify $\int_{-2}^{3}(5-4x)\,dx$.

[6] 13. A rectangle has its base on the x axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest **perimeter** the rectangle can have, and what are its dimensions?