

MATH 249
Worksheet #1 .

For 1 a)

$$|2x + 1| \leq |x - 2|$$

Since $|\dots|$ is always positive or zero we can square both sides and the sign of the inequality stays the same: $(2x + 1)^2 \leq (x - 2)^2$ since $|\dots|^2 = (\dots)^2$

Now

$$4x^2 + 4x + 1 \leq x^2 - 4x + 4 \quad \text{everything on one side: } 3x^2 + 8x - 3 \leq 0$$

$$(3x - 1)(x + 3) \leq 0, \text{ thus split points (roots) are: } x = -3, \frac{1}{3},$$

testing: $\quad \quad \quad \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---}$
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so the solutions set is the closed interval $\left[-3, \frac{1}{3}\right]$

b)

$$\frac{3}{x+1} > \frac{1}{3} \quad \text{for } x \neq -1$$

everything on one side and common denominator: $\frac{3 \cdot 3 - (x+1)}{(x+1)3} > 0$

simplify:

$$\frac{9 - x - 1}{3(x+1)} > 0 \text{ then } \frac{8 - x}{(x+1)(3)} > 0. \quad \text{So split points are: } x = 8, -1$$

testing: $\quad \quad \quad \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---}$
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solution set is the open interval $]-1, 8[$ or $(-1, 8)$.

For 2)

Complete the squares

$$x^2 + 4x + y^2 - 2y = 11 \quad x^2 + 4x + 4 + y^2 - 2y + 1 = 11 + 4 + 1$$

$$(x+2)^2 + (y-1)^2 = 16 \text{ so } (x+2)^2 + (y-1)^2 = 4^2$$

thus

$r = 4$ and the point $C(-2, 1)$ is the centre of the circle.

For 3a) $|x+1| + 2 > 0$

Since $|\dots|$ is always positive or zero $|x+1| + 2$ is always positive for any x , so solution set is $(-\infty, +\infty)$.

b) $\frac{3}{x+1} \geq \frac{2}{x+3} \quad \text{for } x \neq -1, -3$

everything on one side and common denominator: $\frac{3(x+3) - 2(x+1)}{(x+1)(x+3)} \geq 0$

simplify the top:

$$\frac{3x + 9 - 2x - 2}{(x+1)(x+3)} \geq 0 \text{ then } \frac{(x+7)}{(x+1)(x+3)} \geq 0.$$

So split points are: $x = -7, -3, -1$

testing: $\quad \quad \quad \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---}$
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solution set: $[-7, -3] \cup (-1, +\infty)$.

For 4)

For $l_1 : 3x + 2y = 1$ $l_2 : 2y - 3x = 0$ $l_3 : 3x - 2y = 0$ and $l_4 : 2x - 3y = 2$

find the slopes: $m_1 = -\frac{3}{2}$, $m_2 = \frac{3}{2}$, $m_3 = \frac{3}{2}$, $m_4 = \frac{2}{3}$

so $l_2 \parallel l_3$ since they have the same slope
and $l_1 \perp l_4$ since $m_1 \cdot m_4 = -1$.

For 5 a) $\frac{1}{x+1} \leq 1+x$ for $x \neq -1$

everything on one side and common denominator: $\frac{1-(x+1)^2}{(x+1)} \leq 0$, simplify:

$\frac{1-x^2-2x-1}{(x+1)} \leq 0$ then $\frac{-x(x+2)}{(x+1)} \leq 0$. So split points are : $x = 0, -2, -1$
testing $\begin{matrix} -pos & - & -2 & - & -neg & - & -1 & - & -pos & - & - & 0 & - & -neg & - & - \end{matrix}$

solution set: $[-2, -1[\cup [0, +\infty[$ or $[-2, -1) \cup [0, +\infty)$.

b)

$|3x-2| > 0$

Since $|\dots|$ is always positive or zero we have to eliminate zero : $3x-2=0$ for $x = \frac{2}{3}$

The solutions : $x \neq \frac{2}{3}$ or $]-\infty, \frac{2}{3}[\cup]\frac{2}{3}, +\infty[$ or $(-\infty, \frac{2}{3}) \cup (\frac{2}{3}, +\infty)$

For 6)

\perp to x-axis means a vertical line so $x = -1$ (y is any).

line parallel to the x-axis means a horizontal line so $y = 3$ (x any)

For 7 a)

$3x+7 > x^2$

Everything on one side: $0 > x^2 - 3x - 7$ now find the roots ,

first discriminant $D = (-3)^2 - 4 \cdot 1 \cdot (-7) = 9 + 28 = 37$,

so using the quadratic formula roots are

$x_1 = \frac{3-\sqrt{37}}{2} \doteq -1.54$ and $x_2 = \frac{3+\sqrt{37}}{2} \doteq 4.54$

Now testing : $\begin{matrix} - & -pos & - & - & x_1 & - & -neg & - & - & x_2 & - & -pos & - \end{matrix}$

OR parabola open up and it is below the x-axis if $x \in]-1.54, 4.54[$.

(between the roots $x \in (x_1, x_2)$)

b)

$\frac{x}{2} < \frac{2}{x+3}$ for $x \neq -3$

everything on one side and common denominator: $\frac{x(x+3)-2 \cdot 2}{2(x+3)} < 0$

simplify:

$\frac{x^2+3x-4}{2(x+3)} < 0$ then $\frac{(x+4)(x-1)}{2(x+3)} < 0$. So split points are : $x = -4, -3, 1$

testing: $\begin{matrix} -neg & - & -4 & - & -pos & - & -3 & -neg & - & -1 & -pos & - & - \end{matrix}$

solution set: $]-\infty, -4[\cup]-3, 1[$ or $(-\infty, -4) \cup (-3, 1)$.

For 8)

$x^2 - 6x + y^2 = 7$ $x^2 + y^2 + 2y = 15$

Complete the squares :

$x^2 - 6x + 9 + y^2 = 7 + 9$, $x^2 + y^2 + 2y + 1 = 1 + 15$

So the equations are:

$(x-3)^2 + y^2 = 16$ $x^2 + (y+1)^2 = 16$

thus radii are the same $r = 4$, the centres are points $(3, 0)$ and $(0, -1)$.

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For 9) for $h \neq 0, 7$

find the common denominator first, then $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{cb}$

$$\frac{\frac{3h+4}{7-h} - \frac{4}{7}}{\frac{25h}{7}} = \frac{\frac{7(3h+4)-4(7-h)}{7(7-h)}}{\frac{25h}{7}} = \frac{21h+28-28+4h}{7(7-h)} \cdot \frac{7}{25h} = \frac{1}{7-h}$$

For 10) for $x \neq -1$ as in 9)

$$\frac{1}{1 + \frac{1}{x+1}} = \frac{1}{\frac{x+1+1}{x+1}} = \frac{x+1}{x+2} \text{ and for } x \neq -2.$$

For 11)

factor out the polynomials

$$\frac{x^3 + 5x^2 + 6x}{12 + x - x^2} = \frac{x(x^2 + 5x + 6)}{-(x^2 - x - 12)} = \frac{x(x+3)(x+2)}{-(x-4)(x+3)} = -\frac{x(x+2)}{(x-4)}$$

for $x \neq -3, 4$.

For 12)

factor out the polynomials, then common denom.

$$\frac{x}{x^2 + x - 2} - \frac{2}{x^2 - 5x + 4} = \frac{x}{(x+2)(x-1)} - \frac{2}{(x-4)(x-1)} = \frac{x(x-4) - 2(x+2)}{(x+2)(x-1)(x-4)} =$$

$$= \frac{x^2 - 6x - 4}{(x+2)(x-1)(x-4)} \text{ for } x \neq -2, 1, 4.$$

For 13)

$$\frac{x}{x-1} < \frac{1}{x+1} \quad \text{for } x \neq \pm 1$$

everything on one side and common denominator: $\frac{x(x+1) - (x-1)}{(x-1)(x+1)} < 0$

simplify $\frac{x^2 + 1}{(x-1)(x+1)} < 0$ the top has NO real roots, always positive

thus only two split points $x = \pm 1$

Now testing : $- \text{pos} - -_{-1} - \text{neg} - - -_1 - \text{pos} -$

the solution set is $(-1, 1)$.

For 14)

$$\frac{x}{x-1} > \frac{4}{x} \quad \text{for } x \neq 1, 0$$

everything on one side and common denominator: $\frac{x^2 - 4(x-1)}{(x-1)(x)} > 0$

simplify $\frac{x^2 - 4x + 4}{x(x-1)} > 0$ $\frac{(x-2)^2}{x(x-1)} > 0$

the top has a double root, always positive or zero

the split points $x = 0, 1, 2$ but only $x = 0, 1$ are switch points

Now testing :

$- \text{pos} - -_0 - \text{neg} - - -_1 - \text{pos} -_2 - \text{pos} - - -$

check the split points!!

the solution set is $(-\infty, 0) \cup (1, 2) \cup (2, +\infty)$.