MATH 249- 01

Midterm 55 minutes

Fall 2007

Name:	**I.D.#**

NOTE: Calculators are allowed.

1. Find

   (a) $\lim_{x\to 3}\dfrac{\sin(3-x)}{x^2-9}$

   (b) $\lim_{x\to \infty}\dfrac{\sin(3-x)}{x^2-9}$. [7]

2. If $\sin\theta = \frac{3}{7}$and $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$ find $\sec 2\theta$.**NO CALCULATORS**! [6]

3. Find the derivative of $f(x) = \dfrac{\sin(\pi x)}{\sqrt{x}}$ for $x > 0$.Then find the value $f'(4)$. [6]

4. Does the graph of $y = 1 + x - x^4$ cross the $x-$ axis? How many times?

   i.e.Are there real solutions of $1 + x - x^4 = 0$? Find intervals where are the roots located.

   Explain.State the Theorem used. [6]

5. Find the values of a and b so that the function f is continuous everywhere

$$f(x) = \begin{cases} 4\sin^2\dfrac{\pi}{x} & \text{for} \quad x > 4 \\ ax^2 + b & \text{for} \quad 2 \le x \le 4 \\ \cos(2-x) & \text{for} \quad x < 2 \end{cases} .$$ [7]

6. Sketch the graph of ONE function satisfying all the following conditions:

   (a) f is defined on$(-\infty, 2]$

   (b) f is discontinuous at $x = 0, 1$ where $\lim_{x\to 1^+} f(x) = f(1) = 2$, $\lim_{x\to 0} f(x)$ DNE(does not exist),otherwise continuous;

   (c) $x = 1$ is a vertical asymptote and $y = 2$ is a horizontal asymptote;

   (d) f is not differentiable at $x = -1, 0, 1$ (no $f'(-1)$) otherwise differentiable and $f'(x) = 0$ for all $x \in (1, 2)$, also $f'(-2) = 0$;

   (e) f is increasing on $(-2, -1)$,and decreasing on $(-\infty, -2)$ and on $(0, 1)$;

   (f) the maximun value is 3 [8]

SOLUTION

For 1)

$$\lim_{x\to 3}\frac{\sin(3-x)}{x^2-9}=\lim_{x\to 3}\frac{\sin(3-x)}{(x-3)\,(x+3)}=\lim_{x\to 3}\frac{\sin(3-x)}{(3-x)}\cdot\lim_{x\to 3}\frac{1}{-(x+3)}=$$

$$(h=3-x)=\lim_{h\to 0}\frac{\sinh}{h}\cdot\lim_{x\to 3}\frac{1}{-(x+3)}=1\cdot\frac{1}{-6}=-\frac{1}{6}.$$

$\lim_{x\to\infty}\frac{\sin(3-x)}{x^2-9}=0$ by Squ.Theorem

since $\quad -1\le\sin(3-x)\le 1 \qquad \frac{-1}{x^2-9}\le\frac{\sin(3-x)}{x^2-9}\le\frac{1}{x^2-9}$

and $\qquad \lim_{x\to\infty}\frac{const}{x^2-9}=0.$

For 2)

since $\frac{\pi}{2}<\theta<\frac{3\pi}{2}$ and $\sin\theta$ is positive the angle θ is in the second quadrant and

$\cos\theta=-\sqrt{1-\left(\frac{3}{7}\right)^2}=-\sqrt{\frac{40}{49}}=\frac{-2\sqrt{10}}{7}$

then $\csc 2\theta=\frac{1}{\sin 2\theta}=\frac{1}{2\sin\theta\cos\theta}=\frac{1}{2}\cdot\frac{7}{3}\cdot\frac{-7}{2\sqrt{10}}=-\frac{49}{120}\sqrt{10}$

.

For 3)

for $x>0$ by Quotient and Chain rules

$f'(x)=\left[\frac{\sin(\pi x)}{\sqrt{x}}\right]'=\frac{(\sin\pi x)'\sqrt{x}-\sin\pi x\left(\sqrt{x}'\right)}{\left(\sqrt{x}\right)^2}=\frac{\cos\pi x\cdot\pi\cdot\sqrt{x}-\sin\pi x\cdot\frac{1}{2\sqrt{x}}}{x}$

$=\frac{2\pi x\cos\pi x-\sin\pi x}{2x\sqrt{x}}$ $\qquad$ OR by Product Rule

$f'(x)=\left(x^{-\frac{1}{2}}\sin\pi x\right)'=\left(x^{-\frac{1}{2}}\right)'\sin\pi x+x^{-\frac{1}{2}}(\sin\pi x)'=$

$=-\frac{1}{2}x^{-\frac{3}{2}}\sin\pi x+x^{-\frac{1}{2}}(\cos\pi x)(\pi)=\frac{-\sin\pi x}{2x\sqrt{x}}+\frac{\pi\cos\pi x}{\sqrt{x}}$

now at $x=4$ $\qquad f'(4)=\frac{-\sin 4\pi}{8\sqrt{4}}+\frac{\pi\cos 4\pi}{\sqrt{4}}=\frac{\pi}{2}$ since $\sin 4\pi=\sin 0=0,\cos 4\pi=\cos 0=1$

For 4)

the function $f(x)=1+x-x^4$ is continuous everywhere,limits at both ends $-\infty$

: $f(0)=1>0$,together the graph must cross the x-axis **twice**, for x positive and for x negative

check more values $f(-1)=-1<0$

apply Intermediate Value theorem on $[-1,0]$

one value negative ,one positive thus there must be $c\in(-1,0)$

such that the value is 0 $\quad f(c) = 0 \quad c$ is the $x-$intercept (root)

also $\quad f(1) = 1 > 0, f(2) = 3 - 16 < 0$

apply Intermediate Value theorem on $[1, 2]$

one value negative ,one positive thus there must be $c \in (1, 2)$

such that $f(c) = 0 \quad c$ is the $x-$intercept (root)

For 5)

for any values a and b the function f is defined everywhere and it is continuous there except at $x = 4$ and $x = 2$

$$f(x) = \begin{cases} 4\sin^2 \dfrac{\pi}{x} & \text{for} \quad x > 4 \\ ax^2 + b & \text{for} \quad 2 \le x \le 4 \\ \cos(2 - x) & \text{for} \quad x < 2 \end{cases}$$

at $x = 4 \quad f(4) = 16a + b = \lim_{x \to 4^-} f(x)$ and

$$\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} 4\sin^2 \frac{\pi}{x} = 4\left[\sin\left(\frac{\pi}{4}\right)\right]^2 = 4 \cdot \frac{1}{2} = 2$$

so it must $\quad 16a + b = 2$

at $x = 2 \quad f(2) = 4a + b = \lim_{x \to 2^+} f(x)$ and

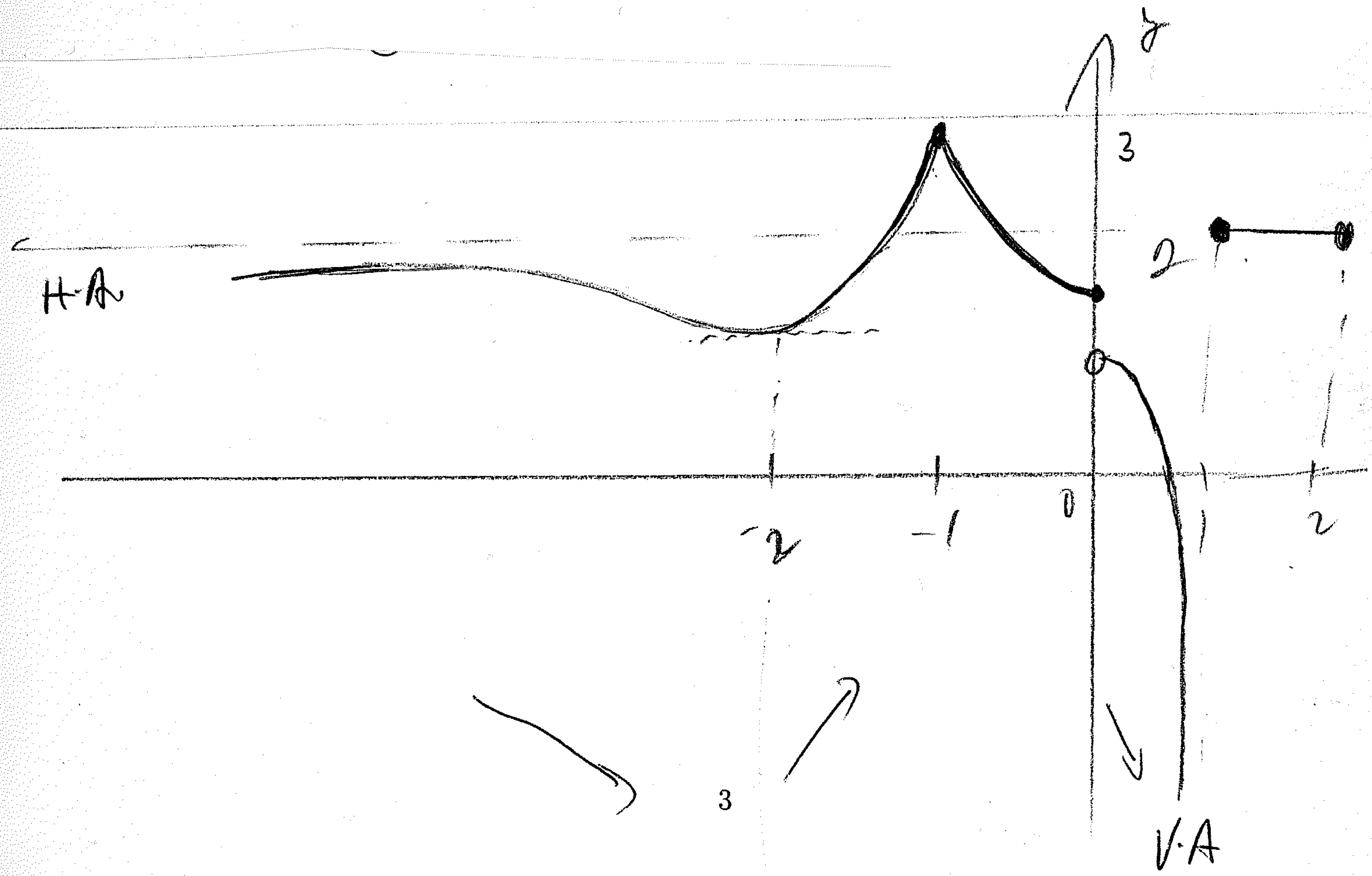
$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \cos(2 - x) = \cos 0 = 1 \quad$ so $\quad 4a + b = 1$

now,solve the system $\quad 16a + b = 2 \quad 4a + b = 1$

subtract the equations to get $\quad 12a = 1$

so $a = \dfrac{1}{12}$ and $b = 1 - 4a = \dfrac{2}{3}$.

For 6)



y
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