

MATH 249
Midterm Handout

1. Evaluate

$$\lim_{x \rightarrow \infty} \left(x^2 - x^2 \cos \frac{1}{x} \right)$$

2. Evaluate

$$(a) \quad \lim_{x \rightarrow 0} \frac{\sin x}{x - \pi} \quad (b) \quad \lim_{x \rightarrow \pi} \frac{\sin x}{x - \pi} \quad (c) \quad \lim_{x \rightarrow -\infty} \frac{\sin x}{x - \pi}.$$

3. For $y = \frac{\cos \pi x}{1 - x}$ find an equation of the tangent line at $x = -\frac{1}{2}$.

4. For $y = \left(\sin \frac{1}{\sqrt{x^4 + 1}} \right)^3$ find y' .

5. Show that the function $f(x) = x - 2 \sin(\pi x)$ has at least one positive zero i.e. $f(x) = 0$ at least for one $x > 0$.

6. Locate all 3 roots of $p(x) = 2x^3 - 6x^2 + 7$ i.e. find 3 intervals each containing one root. Sketch the graph of $y = p(x)$.

7. Find $\sec \theta$ if $\sin \theta = \frac{1}{5}$ and $\frac{\pi}{2} < \theta < \frac{3}{2}\pi$.

8. If $\cos \theta = \frac{2}{3}$ and $\pi < \theta < 2\pi$ find $\sin \theta$ and then $\sin 2\theta$.

9. Find the values of a and b so that the function f is continuous everywhere

$$f(x) = \begin{cases} \left(\frac{2}{2x+1} - 3 \right) (4x^2 - 1) & \text{for } x < -\frac{1}{2} \\ ax + b & \text{for } -\frac{1}{2} \leq x \leq 2 \\ \cos\left(-\frac{\pi}{x}\right) & \text{for } x > 2 \end{cases}.$$

10. Find the values of a and b so that the function f is continuous everywhere

$$f(x) = \begin{cases} \cos(\pi x) - 2 \sin \frac{\pi x}{2} & \text{for } x > 3 \\ ax^2 + b & \text{for } 0 \leq x \leq 3 \\ 6 \cdot \frac{\sqrt{9-x} - 3}{x} & \text{for } x < 0 \end{cases}.$$

11. **A**

Sketch the graph of ONE function satisfying all the following conditions:

(a) f is defined on $[-2, +\infty)$

(b) f is discontinuous at $x = 0, 1, 2$ where $\lim_{x \rightarrow 1} f(x) = 3$, $\lim_{x \rightarrow 2} f(x)$ DNE (does not exist). otherwise continuous

(c) $x = 0$ is a vertical asymptote and $y = -2$ is a horizontal asymptote

(d) f is not differentiable at $x = -1, 0, 1, 2$ (no $f'(-1)$) otherwise differentiable and $f'(x) = 0$ for all $x \in (0, 1)$, also $f'(4) = 0$.

- (e) f is increasing on $(-2, -1)$ and on $(2, 4)$; decreasing on $(-1, 0)$ and on $(4, +\infty)$;
- (f) the maximum value is 3.

B

Sketch the graph of ONE function satisfying all the following conditions:

- (a) f is defined on $(-\infty, 1]$
- (b) f is discontinuous at $x = -3$ and $x = -2$ where $\lim_{x \rightarrow -3^+} f(x) = f(-3) = 5$ otherwise continuous
- (c) $x = -2$ is a vertical asymptote and $\lim_{x \rightarrow -\infty} f(x) DNE$ (does not exist)
- (d) f is not differentiable at $x = -1, -2, -3$ (no $f'(-1)$) otherwise differentiable and $f'(x) = 0$ for all $x \in (-1, 0)$, also $f'(-4) = 0$;
- (e) f is increasing on $(-3, -2)$ and on $(0, 1)$; decreasing on $(-2, -1)$;
- (f) the minimum value is -2 .

C

Sketch the graph of ONE function satisfying all the following conditions:

- (a) f is defined on $(-\infty, 2)$
- (b) f is discontinuous at $x = -3$ and $x = -2$ where $\lim_{x \rightarrow -3} f(x) = 2$, and $x = -2$ is a vertical asymptote, otherwise continuous
- (c) $y = 1$ is a horizontal asymptote
- (d) f is not differentiable at $x = -1, -2, -3$ (no $f'(-1)$) otherwise differentiable and $f'(x) = 0$ for all $x \in (1, 2)$, also $f'(-4) = 0$;
- (e) f is increasing on $(-1, 1)$ and on $(-3, -2)$; decreasing on $(-2, -1)$;
- (f) the minimum value is $\frac{1}{2}$.

D

Sketch the graph of ONE function satisfying all the following conditions:

- (a) f is defined on $(-1, \infty)$
- (b) f is discontinuous at $x = 3$ and $x = 2$ where $\lim_{x \rightarrow 2^+} f(x) = f(2) = 3$, $x = 3$ is a vertical asymptote, otherwise continuous
- (c) and $\lim_{x \rightarrow +\infty} f(x) DNE$ (does not exist)
- (d) f is not differentiable at $x = 0, 2, 3$ (no $f'(0)$) otherwise differentiable and $f'(x) = 0$ for all $x \in (-1, 0)$, also $f'(4) = 0$.
- (e) f is increasing on $(2, 3)$ and on $(3, 4)$; decreasing on $(0, 2)$;

E

Sketch the graph of ONE function satisfying all the following conditions:

- (a) f is defined on $(-\infty, +\infty)$
- (b) f is discontinuous at $x = -1$ and $x = 2$ where $\lim_{x \rightarrow -1} f(x)$ DNE
 $x = 2$ is a vertical asymptote, otherwise continuous
- (c) and $\lim_{x \rightarrow -\infty} f(x)$ DNE (does not exist), $y = -3$ is a horizontal asymptote;
- (d) f is not differentiable at $x = -1, 1, 2$ (no $f'(1)$) otherwise differentiable
and $f'(x) = 0$ for all $x \in (2, 3)$, also $f'(4) = 0$;
- (e) f is increasing on $(-1, 1)$; decreasing on $(1, 2)$ and on $(4, +\infty)$
- (f) the maximum value is 4.