The University of Calgary
Department of Mathematics and Statistics
MATH 249-01
Quiz # 2R

Fall 2007

Name:____________________________**I.D.#:**____________________

1. For $f(x) = \dfrac{5x}{x-3}$ and $g(x) = \sqrt{4-x}$ find the composition $g \circ f$ and the domain:$D_f, D_g, D_{g \circ f.}$. [5]

2. For $f(x) = \dfrac{1}{x-2}\left(\dfrac{1}{x} - \dfrac{x}{4}\right)$ find $\lim f(x)$
(a) as $x \to 2$;(b)as $x \to +\infty$; (c) as $x \to 0^+$ [5]

3. For $g(x) = \dfrac{2-\sqrt{x+3}}{x-1}$ find $\lim g(x)$
(a) as $x \to 1$; (b) as $x \to -\infty$ (c) as $x \to +\infty$. [5]

SOLUTION

For 1)

$f(x) = \dfrac{5x}{x-3}$ and $g(x) = \sqrt{4-x}$ $\qquad D_f = \{x \neq 3\}; D_g = (-\infty, 4]$

$$(g \circ f)(x) = g(...) = \sqrt{4 - \frac{5x}{x-3}} = \sqrt{\frac{4x-12-5x}{x-3}} = \sqrt{\frac{-x-12}{x-3}}$$

for domain solve $\quad \dfrac{-x-12}{x-3} \geq 0 \qquad x = -12, 3$ split points

testing $\qquad - -^{neg} - - -_{-12} -^{pos} - -_{3} -^{neg} -$

$D_{g \circ f} = [-12, 3)$

For 2)

for a) first we can simplify for $x \neq 0, 2$

$$f(x) = \frac{1}{x-2}\left(\frac{1}{x} - \frac{x}{4}\right) = \frac{1}{x-2} \cdot \frac{4-x^2}{4x} = \frac{(2-x)(2+x)}{4x(x-2)} = \frac{-(2+x)}{4x}$$

$$\lim_{x \to 2} f(x) = \tfrac{-4}{8} = -\frac{1}{2}.$$

for b)

$$\lim_{x \to +\infty} f(x) = \lim_{x \to -\infty} \frac{-(2+x)}{4x} = \lim_{x \to +\infty} \frac{-1}{2x} - \frac{1}{4} = -\frac{1}{4} (\text{since } "\tfrac{1}{\infty}" = 0)$$

also from the original

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{4-x^2}{4x(x-2)} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \text{"} \lim_{x\to+\infty} \frac{\frac{4}{x}-1}{4\left(1-\frac{2}{x}\right)} = \frac{-1}{4}.$$

for c)

$$\lim_{x\to0^+} f(x) = \lim_{x\to0^+} \frac{-(2+x)}{4x} = \text{"}\frac{-2}{0^+}\text{"} = -\infty$$ or from the original

$$\lim_{x\to0^+} \frac{1}{x-2}\left(\frac{1}{x}-\frac{x}{4}\right) = \frac{-1}{2}\left(\text{"}\frac{1}{0^+}\text{"}-0\right) = -\infty.$$

For 3 a)

g is defined for $x \in (-\infty, -3]$ and $x \neq 1$ since $(x+3) \geq 0$

$$g(x) = \frac{2-\sqrt{x+3}}{x-1} \cdot \frac{2+\sqrt{x+3}}{2+\sqrt{x+3}} = \frac{4-(x+3)}{(x-1)\left(2+\sqrt{x+3}\right)} = \frac{-(x-1)}{(x-1)\left(2+\sqrt{x+3}\right)} = \frac{-1}{\left(2+\sqrt{x+3}\right)}$$

and $\lim_{x\to1} g(x) = \lim_{x\to1} \frac{-1}{\left(2+\sqrt{x+3}\right)} = \frac{-1}{4}$

for b) we can use the original or simplified form since x is big negative

g is not defined since $\sqrt{neg}$ no values

the limit DNE= does not exist

for c) we can use the original form since x is big positive,so $x = \sqrt{x^2}$

type is $\text{"}\frac{-\infty}{\infty}\text{"}$

$$\lim_{x\to+\infty} \frac{2-\sqrt{x+3}}{x-1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\frac{2}{x}-\frac{\sqrt{x+3}}{x}}{1-\frac{1}{x}} = \lim_{x\to\infty} \frac{\frac{2}{x}-\frac{\sqrt{x+3}}{\sqrt{x^2}}}{1-\frac{1}{x}} = \lim_{x\to\infty} \frac{\frac{2}{x}-\sqrt{\frac{x+3}{x^2}}}{1-\frac{1}{x}} =$$

$$= \lim_{x\to-\infty} \frac{\frac{2}{x}-\sqrt{\frac{1}{x}+\frac{3}{x^2}}}{1-\frac{1}{x}} = \frac{0}{1} = 0$$

OR the simplified from $\lim_{x\to+\infty} \frac{-1}{\left(2+\sqrt{x+3}\right)} = \text{"}\frac{-1}{\infty}\text{"} = 0$