The University of Calgary
Department of Mathematics and Statistics
MATH 249-01
Quiz # 2W

Fall 2007

Name:______________________________**I.D.#:**__________________

1. For $f(x) = \dfrac{1}{x-3}$ and $g(x) = \sqrt{x+1}$ find the compositions
   $f \circ g$ and the domains:$D_f, D_g, D_{f \circ g}.$ [5]

2. For $g(x) = \dfrac{|x-3|}{x^2-4x+3}$ find $\lim g(x)$
   (a) as $x \to 3^-$; (b) as $x \to 1^+$ (c) as $x \to +\infty$. [5]

3. For $f(x) = \dfrac{1-(2x-1)^2}{x^2+x}$ find $\lim f(x)$
   (a) as $x \to 0$; (b)as $x \to -\infty$ (c) as $x \to -1^+$. [5]

SOLUTION

For 1)

First domain of the given functions

for $f(x) = \dfrac{1}{x-3}$ it must $x - 3 \neq 0$ $D_f = \{x \neq 3\}$ and

for $g(x) = \sqrt{x+1}$ it must $x + 1 \geq 0 \qquad x \geq -1 \qquad D_g = [-1, \infty)$

for $x \geq -1$

$$(f \circ g)(x) = f(...) = \frac{1}{() - 3} = \frac{1}{\sqrt{x+1} - 3} \cdot \frac{\sqrt{x+1}+3}{\sqrt{x+1}+3} = \frac{\sqrt{x+1}+3}{x-8}$$

OR $\qquad \sqrt{x+1} - 3 \neq 0 \qquad \sqrt{x+1} \neq 3 \qquad x \neq 9 - 1$

so $x \neq 8$ together $D_{f \circ g} = [-1, 8) \cup (8, +\infty)$

For 2a)

first $x < 3 \qquad x - 3 < 0 \qquad |x-3| = -(x-3)$

$$g(x) = \frac{|x-3|}{x^2-4x+3} = \frac{-(x-3)}{(x-3)(x-1)} = \frac{-1}{x-1} \text{ thus}$$

$$\lim_{x \to 3^-} g(x) = \lim_{x \to 3^-} \frac{-1}{x-1} = -\frac{1}{2}$$

for b)

for $x > 1$

$$\lim_{x \to 1^+} g(x) = \lim_{x \to 1^+} \frac{|x-3|}{(x-3)(x-1)} = \text{"} \frac{2}{-2 \cdot 0^+} \text{"} = \text{"} \frac{1}{0^-}\text{'} = -\infty$$

also we can use the simplied from from part a) since x is close to 1 thus $x < 3$

For c)

$$\lim_{x\to+\infty} g(x) = \lim_{x\to+\infty} \frac{|x-3|}{x^2-4x+3} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to+\infty} \frac{\left|\frac{x-3}{x^2}\right|}{1-\frac{4}{x}+\frac{3}{x^2}} = \lim_{x\to+\infty} \frac{\left|\frac{1}{x}-\frac{3}{x^2}\right|}{1-\frac{4}{x}+\frac{3}{x^2}} = \frac{0}{1} = 0$$

OR since x is big positive $|x-3| = x-3$ and

$$g(x) = \frac{(x-3)}{(x-3)(x-1)} = \frac{1}{x-1}$$

and $\lim_{x\to+\infty} g(x) \acute{=} \frac{1}{\infty}" = 0$

For 3)

for $x \neq 0, -1$

$$f(x) = \frac{1-(2x-1)^2}{x^2+x} = \frac{1-(4x^2-4x+1)}{x(x+1)} = \frac{-4x(x-1)}{x(x+1)} = \frac{-4(x-1)}{(x+1)}$$

for a)

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{-4(x-1)}{(x+1)} = 4$$

for b)

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{1-(2x-1)^2}{x^2+x} = \lim_{x\to+\infty} \frac{-4x^2+4x}{x^2+x} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to+\infty} \frac{-4+\frac{4}{x}}{1+\frac{1}{x}} = -4$$

for c)

$$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} \frac{-4(x-1)}{(x+1)} = " \frac{8}{0^+} " = +\infty$$