The University of Calgary
Department of Mathematics and Statistics
MATH 249-01
Quiz #3W

FALL 2007

Name:______________________________**I.D.#:**____________________

1. Using the **definition of derivative** find $f'(-1)$ if $f(x) = \dfrac{2x}{3+x}$. [5]

2. Find y' if $y = \sqrt{x^2+1}(4+\dfrac{1}{2x})$ for $x \neq 0$. [5]

3. Find an equation of the tangent line to $y = \dfrac{2\sqrt{x}}{2-x}$ at $x = 4$. [5]

Solution
For 1)

$$f'(-1) = \lim_{x\to -1}\frac{f(x)-f(-1)}{x-(-1)} = \lim_{x\to -1}\frac{\frac{2x}{3+x}-\frac{-2}{2}}{x+1} = \lim_{x\to -1}\frac{\frac{2x}{3+x}+1}{x+1} =$$

$$= \lim_{x\to -1}\frac{\frac{2x+3+x}{3+x}}{x+1} = \lim_{x\to -1}\frac{\frac{3(x+1)}{3+x}}{x+1} = \lim_{x\to -1}\frac{3(x+1)}{(3+x)(x+1)} = \lim_{x\to -1}\frac{3}{x+3} = \frac{3}{2}$$

$$f'(-1) = \lim_{h\to 0}\frac{f(-1+h)-f(-1)}{h} = \lim_{h\to 0}\frac{\dfrac{2(-1+h)}{3+(-1+h)}+1}{h} =$$

$$= \lim_{h\to 0}\frac{\dfrac{2h-2}{2+h}+1}{h} = \lim_{h\to 0}\frac{1}{h}\left[\frac{2h-2+2+h}{2+h}\right] =$$

$$= \lim_{h\to 0}\tfrac{1}{h}\cdot\frac{3h}{2+h} = \lim_{h\to 0}\frac{3}{2+h} = \frac{3}{2}$$

(Check by rules $f'(x) = 2\cdot\dfrac{3+x-x}{(3+x)^2} = \dfrac{6}{(3+x)^2}$ at $x=-1$ $f'(-1) = \frac{6}{4} = \frac{3}{2}$)

also

For 2)

use Product and Chain Rules

$$y = \left[(x^2+1)^{\frac{1}{2}}\right]'(4+\frac{1}{2x}) + \sqrt{x^2+1}(4+\frac{1}{2}x^{-1})' =$$

$$= \tfrac{1}{2}(x^2+1)^{-\frac{1}{2}}\cdot(x^2+1)'(4+\frac{1}{2x}) + \sqrt{x^2+1}(0+\frac{1}{2}(-1)x^{-2}) =$$

so $\quad y' = \tfrac{1}{2}(x^2+1)^{-\frac{1}{2}}\cdot 2x(4+\dfrac{1}{2x}) + \sqrt{x^2+1}(\dfrac{-1}{2x^2}) = \dfrac{4x+\frac{1}{2}}{\sqrt{x^2+1}} - \dfrac{\sqrt{x^2+1}}{2x^2}$

.

For 3)

at $x=4$ $\quad y = \frac{4}{-2} = -2$ the point is $P(4,-2)$ and the line through P is
$y = -2 + m(x-4)$

slope of a tangent is given by $y' = \left(\dfrac{2\sqrt{x}}{2-x}\right)'$ at $x = 4$

we can use Quotient Rule

$$y' = \frac{(2\sqrt{x})'\,(2-x) - 2\sqrt{x}\,(2-x)'}{(2-x)^2} = \frac{2\cdot\frac{1}{2\sqrt{x}}\,(2-x) - 2\sqrt{x}\,(-1)}{(2-x)^2}$$

at $x = 4$

$$m = \frac{\frac{1}{\sqrt{4}}\,(-2) + 2\sqrt{4}}{(-2)^2} = \frac{7}{4} \qquad y = -2 + \frac{7}{4}(x-4)$$

or $\quad y = \frac{7}{4}x - 9 \quad$ *or* $\quad 7x - 4y = 36$