The University of Calgary
Department of Mathematics and Statistics
MATH 249- 01 Quiz #3R

FALL 2007

Name:______________________________**I.D.#:**__________________]

1. Using the **definition of the derivative** find $f'(-2)$ if $f(x)\sqrt{8-\frac{x}{2}}$. [5]

2. Find $f'(x)$ if $f(x)=\left(\frac{x^3}{3}+\frac{1}{2x^2}\right)\cdot(5+2x)^3$ for $x\neq 0$. [5]

3. Find all points on the graph $y=\sqrt{4x^2+6x+3}$ with horizontal tangents.Find equations of the tangent lines. [5]

For 1) $f(2)=\sqrt{8+1}=3$

$$f'(-2) = \lim_{x\to-2}\frac{f(x)-f(-2)}{x+2} = \lim_{x\to-2}\frac{\sqrt{8-\frac{x}{2}}-3}{x+2}\cdot\frac{\sqrt{8-\frac{x}{2}}+3}{\sqrt{8-\frac{x}{2}}+3} = \lim_{x\to-2}\frac{8-\frac{x}{2}-3^2}{x+2}\cdot\frac{1}{\sqrt{8-\frac{x}{2}}+3} =$$

$$= \lim_{x\to-2}\frac{-\frac{1}{2}(2+x)}{(x+2)\left(\sqrt{8-\frac{x}{2}}+2\right)} = \lim_{x\to3}\frac{-\frac{1}{2}}{\sqrt{8-\frac{x}{3}}+3} = -\frac{1}{12} \qquad \text{OR}$$

$$f'(-2)=\lim_{h\to0}\frac{f(-2+h)-f(-2)}{h}=\lim_{h\to0}\frac{\sqrt{8-\frac{-2+h}{2}}-3}{h}=\lim_{h\to0}\frac{\sqrt{\frac{18-h}{2}}-3}{h}\cdot\frac{\sqrt{\frac{18-h}{2}}+3}{\sqrt{\frac{18-h}{2}}+3}=$$

$$=\lim_{h\to0}\frac{\frac{18-h}{2}-9}{h\left(\sqrt{\frac{18-h}{2}}+3\right)}=\lim_{h\to0}\frac{-\frac{h}{2}}{h\left(\sqrt{\frac{18-h}{2}}+3\right)}=\frac{-\frac{1}{2}}{3+3}=-\frac{1}{12}.$$

Check by Rules: $f'(x)=\frac{1}{2}\left(8-\frac{x}{2}\right)^{-\frac{1}{2}}\cdot\left(-\frac{1}{2}\right)$ and at $x=-2$,

$f'(-2)=\frac{-1}{4}\cdot(8+1)^{-\frac{1}{2}}=-\frac{1}{4}\cdot\frac{1}{\sqrt{9}}=-\frac{1}{12}$.

For 2) For $x\neq0$ by Product Rule

$$f'(x)=\left(\frac{1}{3}x^3+\frac{1}{2}x^{-2}\right)'(5+2x)^3+\left(\frac{x^3}{3}+\frac{1}{2x^2}\right)\left[(5+2x)^3\right]'=$$

and by Power and Chain Rules

$$=\left(\frac{1}{3}\cdot3x^2+\frac{1}{2}(-2)x^{-3}\right)(5+2x)^3+\left(\frac{x^3}{3}+\frac{1}{2x^2}\right)\cdot3\,(5+2x)^2\,(5+2x)'=$$

$$=\left(x^2-\frac{1}{x^3}\right)(5+2x)^3+\left(\frac{x^3}{3}+\frac{1}{2x^2}\right)6\,(5+2x)^2=....$$

For 3)

horizontal tangent means $m=y'=0$, by Chain Rule

$$y'=(\sqrt{4x^2+6x+3})'=\frac{1}{2}\left(\sqrt{4x^2+6x+3}\right)^{-\frac{1}{2}}\cdot(4x^2+6x+3)'=\frac{8x+6}{2\sqrt{4x^2+6x+3}}$$

$y' = 0$ if the numerator is 0: $\quad 8x + 6 = 0 \quad x = -\frac{6}{8} = -\dfrac{3}{4}$

For $x = \frac{-3}{4}$, $y = \sqrt{\frac{9}{4} - \frac{9}{2} + 3} = \dfrac{\sqrt{3}}{2}$,so the point is $P\left(-\dfrac{3}{4}, \dfrac{\sqrt{3}}{2}\right)$

and the equation is $\quad y = \dfrac{\sqrt{3}}{2}$.