

The University of Calgary  
 Department of Mathematics and Statistics  
 MATH 249-01  
 Quiz # 4R

FALL 2007

Name: \_\_\_\_\_ I.D.#: \_\_\_\_\_

1. Find an equation of the tangent to

$$\sqrt{2x + y} = 6 + \frac{xy}{6}$$

at the point  $(6, -3)$ . [5]

2. Find the second derivative of  $f(x) = \frac{1}{3x^2 + 5}$ . [5]

3. Find a general antiderivative of  $f(x) = \sqrt{x}(3x - \frac{2}{\sqrt{x}}) + \cos(2x)$  for  $x > 0$ .  
 Simplify first. [5]

**Solution**

**For 1)**

an equation  $y = m(x - 6) - 3$  and to find  $m$  use

implicit differentiation  $[(2x + y)^{\frac{1}{2}}]' = 0 + \frac{1}{6}(xy)'$

$$\frac{1}{2}(2x + y)^{-\frac{1}{2}} \cdot (2x + y)' = \frac{1}{6}(1 \cdot y + xy') \quad \text{multiply by 6}$$

$$3(2x + y)^{-\frac{1}{2}}(2 + y') = y + xy' \quad \text{now, } x = 6, y = -3, y' = m$$

$$\frac{3}{\sqrt{9}}(2 + m) = -3 + 6m \text{ so } 2 + m = -3 + 6m$$

$$\text{and } 5 = 5m, m = 1 \quad y = (x - 6) - 3 \text{ OR } y = x - 9$$

**For 2)**

$$f(x) = \frac{1}{3x^2 + 5} = (3x^2 + 5)^{-1}$$

by Chain rule or Quotient rule

$$f'(x) = -(3x^2 + 5)^{-2}(3x^2 + 5)' = -6x(3x^2 + 5)^{-2}$$

by Chain and Product rules

$$f''(x) = -6(3x^2 + 5)^{-2} - 6x(-2)(3x^2 + 5)^{-3}(6x) = -6(3x^2 + 5)^{-2} + 72x^2(3x^2 + 5)^{-3}$$

**For 3)**

$$f(x) = \sqrt{x}(3x - \frac{2}{\sqrt{x}}) + \cos(2x) = 3x^{\frac{3}{2}} - 2 + \cos(2x)$$

$$\int f(x)dx = 3 \int x^{\frac{3}{2}}dx - 2 \int dx + \int \cos(2x)dx = 3 \cdot \frac{2}{5}x^{\frac{5}{2}} - 2x + \frac{1}{2} \sin(2x) + c$$

$$= \frac{6}{5}x^{\frac{5}{2}} - 2x + \frac{1}{2} \sin(2x) + c$$

$$\text{using } \int \cos(ax + b)dx = \frac{1}{a} \sin(ax + b) + c, a = 2, b = 0$$